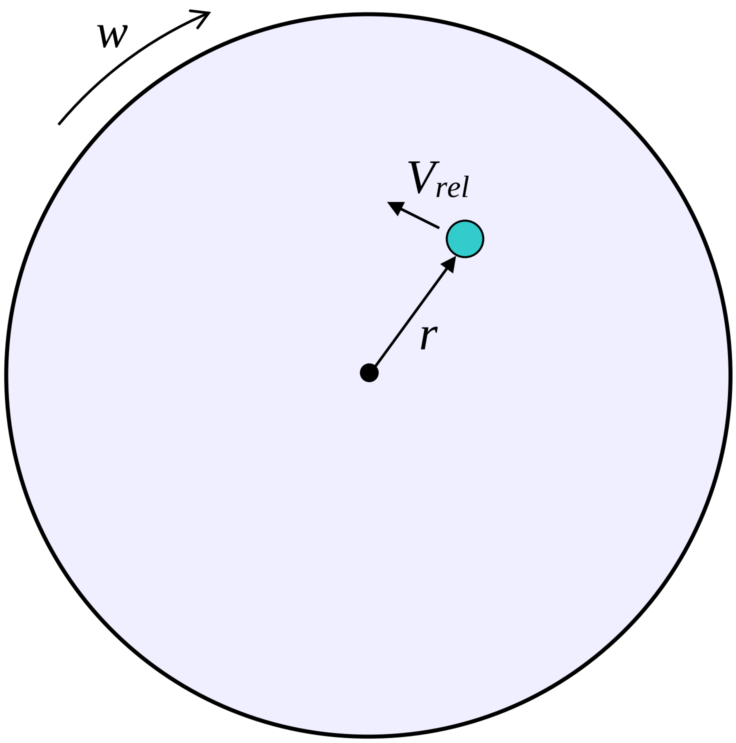
Problem

A child on a horizontal merry-go-round (shown below) gives an initial velocity V_{rel} to a ball. Find the initial direction and velocity V_{rel} of the ball relative to the merry-go-round so that, relative to the child, the ball goes around in a perfect circle as he's sitting on the merry-go-round. Assume there is no friction between merry-go-round and ball.

The merry-go-round is rotating at a constant angular velocity of *w* radians/second, and the ball is released at a radius *r* from the center of the merry-go-round.



Solution

Since there is no friction between the ball and merry-go-round, the general motion of the ball is such that it will either be stationary or travel in a straight line relative to fixed ground (in other words, as observed by someone standing on the ground). This happens because there are no net external forces acting on the ball (Newton's First Law).

However, relative to the child sitting on the merry-go-round, the motion of the ball follows a curved path. We want to find the initial velocity and direction of the ball such that the path traced on the merry-go-round (by the ball) is a perfect circle.

A perfect circle means that the ball returns to its starting point after one full rotation of the merry-go-round. This means that the ball must have a velocity of zero relative to fixed ground. The ball will appear to be a stationary dot (relative to ground) coinciding with the initial location of the child. As the merry-go-round turns, the child sees this "dot" moving away from him. And after one full revolution he is once more coincident with the "dot".

Therefore, in order to have zero velocity of the ball relative to fixed ground, the child must roll the ball away from him at an angle of 90°, opposite the direction of rotation of the merry-go-round, and with a relative speed equal to $V_{rel} = wr$ (see figure below). This will cancel out the tangential velocity of the merry-go-round at the location of the ball (equal to wr), and the resulting velocity of the ball will be zero relative to fixed ground.