# User Manual For Trebuchet Simulator In Excel

**© Franco Normani**

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**Email: franco@real-world-physics-problems.com**

**Alternate email: fvnorman@hotmail.com**

**Legal Notice and Disclaimer**

This Excel program and manual are for personal use only. You are not permitted to sell or redistribute this program and manual in any way.

The results of the program are accurate as far as the physics and mathematics are concerned. But you are still expected to exercise good judgment when designing and building a trebuchet. Therefore, I am not responsible for the use or misuse of the program, or the information presented here.

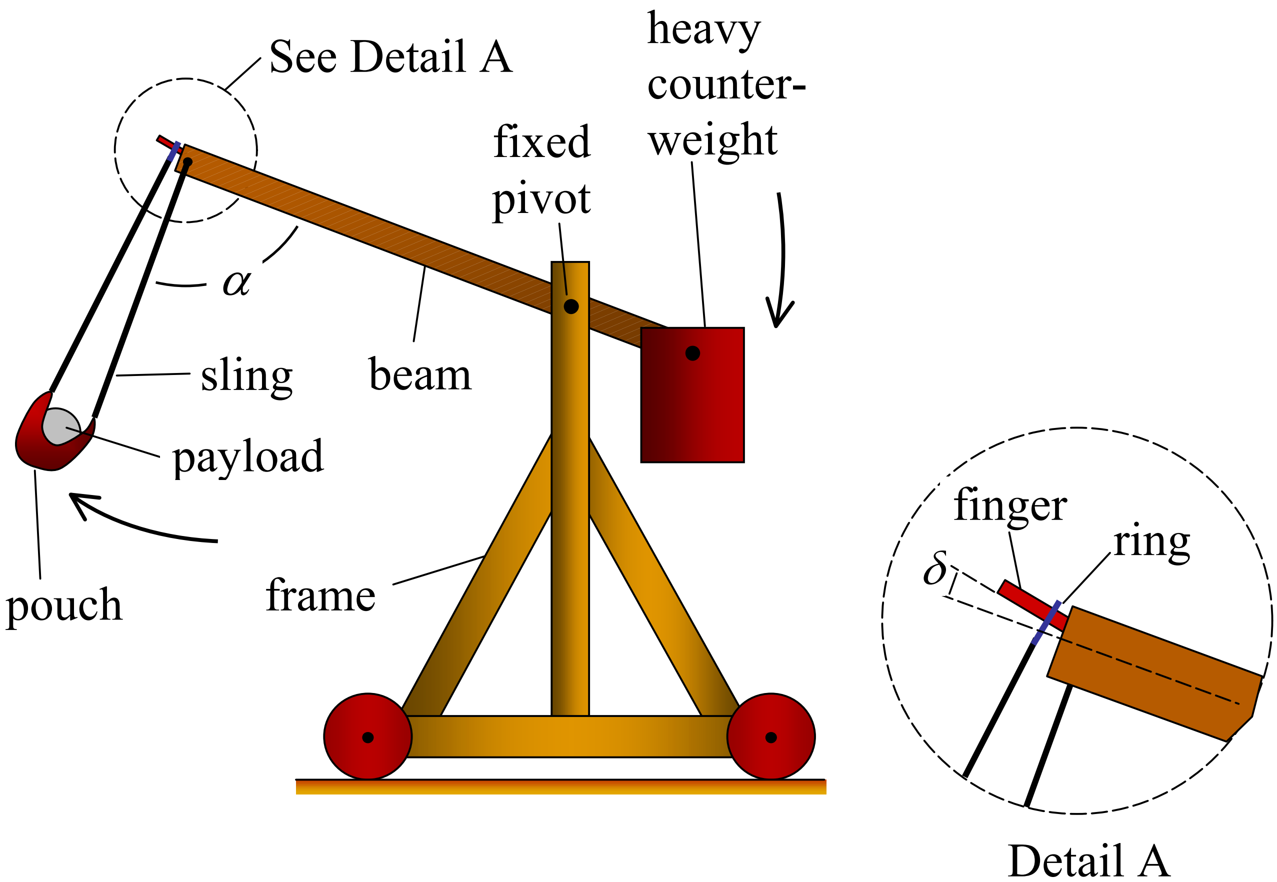
**Purpose**

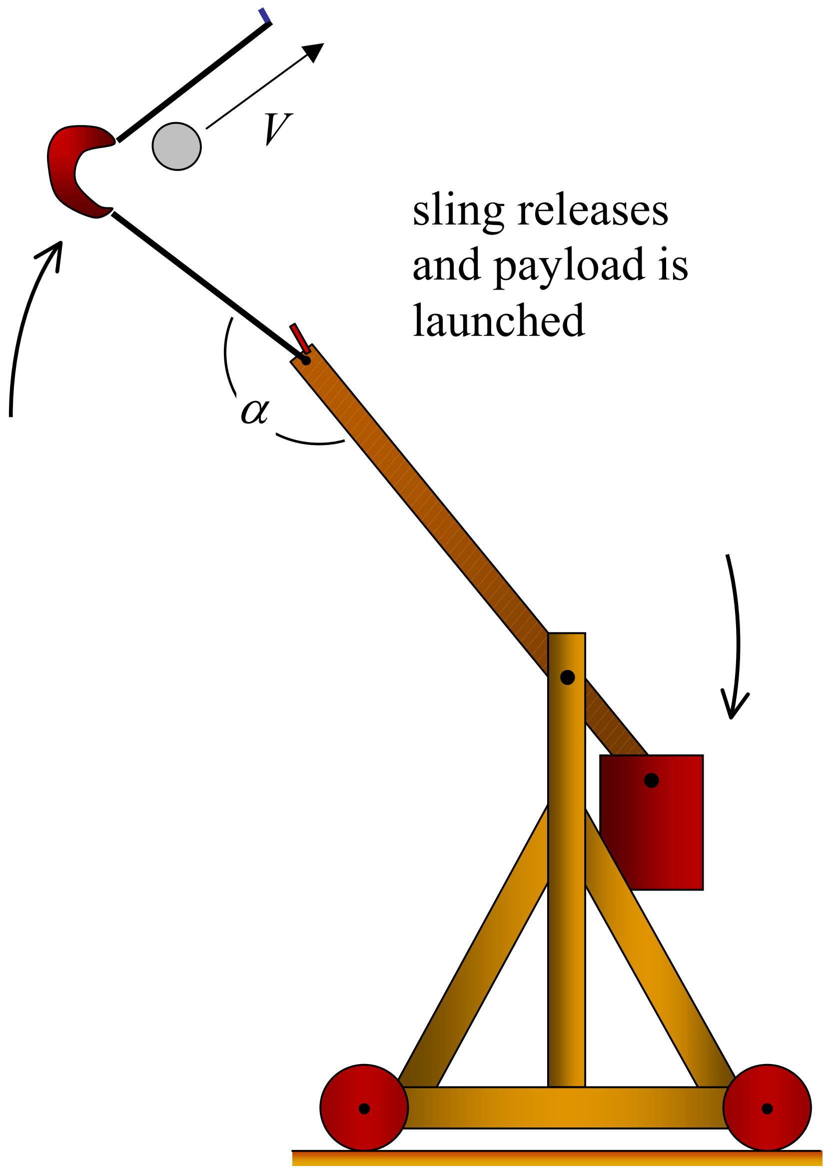
The purpose of this Excel program (spreadsheet) and manual is to allow you to optimize the dimensions of a trebuchet to give you the farthest throwing distance. The program and manual are not intended as a material selection guide, or a construction guide. However, the information given in the Excel program will give you all the information you need to calculate things like material stress, material type, and the size of components necessary to withstand the loads experienced by the trebuchet when "in service".

Remember to exercise caution if you decide to build and test a trebuchet, since they are capable of doing serious damage; something the ancients knew all too well.

**Trebuchet Description**

The two labeled figures below show a trebuchet during launching of the payload.






If there is no friction on the finger, the ring (attached to the sling) begins to slide off the finger when $\alpha = 90° + \delta$. At this point the sling releases and the payload is launched. (See page 12 for the case where there is friction between the ring and finger).

**Assumptions and Analysis**

The following assumptions are made in the model:

- The trebuchet is rigid. There is no flexing of the various members.
- There is no friction on the guide chute (discussed below) or at the joints (pivots).
- The sling and pouch have negligible mass.
- There is no air resistance as the payload flies through the air.
- The trebuchet remains stationary on the ground during launch.

Figure 1 below (with sign convention and dimensions labeled, as shown) shows a simplified schematic of the trebuchet, where a single cable models the connection between the counterweight and beam, and the connection between the payload and beam (the sling). This figure shows the (assumed) <u>initial</u> start position of the launch when a guide chute is used (**keep this in mind when using the Excel spreadsheet**). A guide chute is used to guide the sling along and support the payload until the speed is great enough to hold it in the pouch alone.

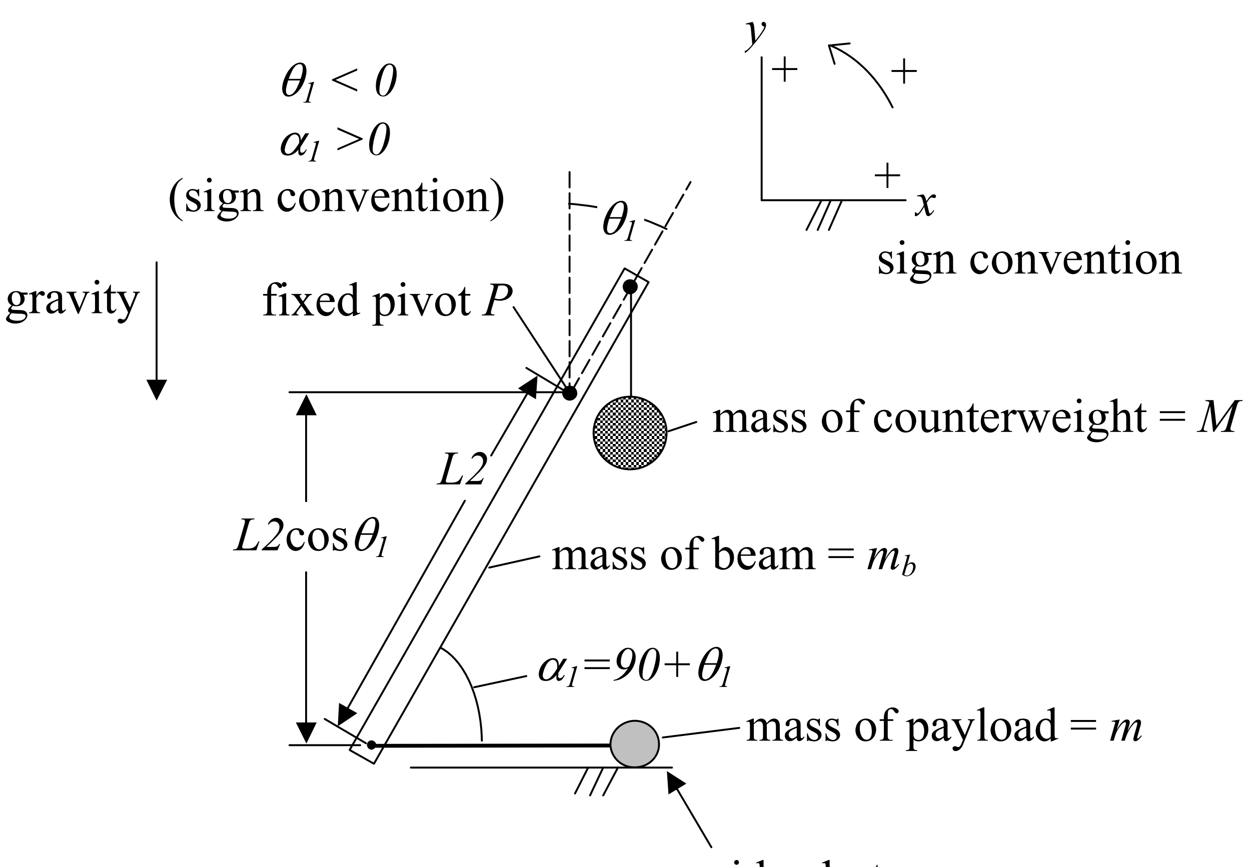



Figure 1

Figure 2 below (with sign convention and dimensions labeled, as shown) shows the trebuchet after release, as the payload is sliding along the guide chute. This is considered to be a constrained case since the guide chute "holds" the payload in place as it slides along. The instant the payload loses contact with the guide chute we have an unconstrained situation (Case 2, shown on the next page).

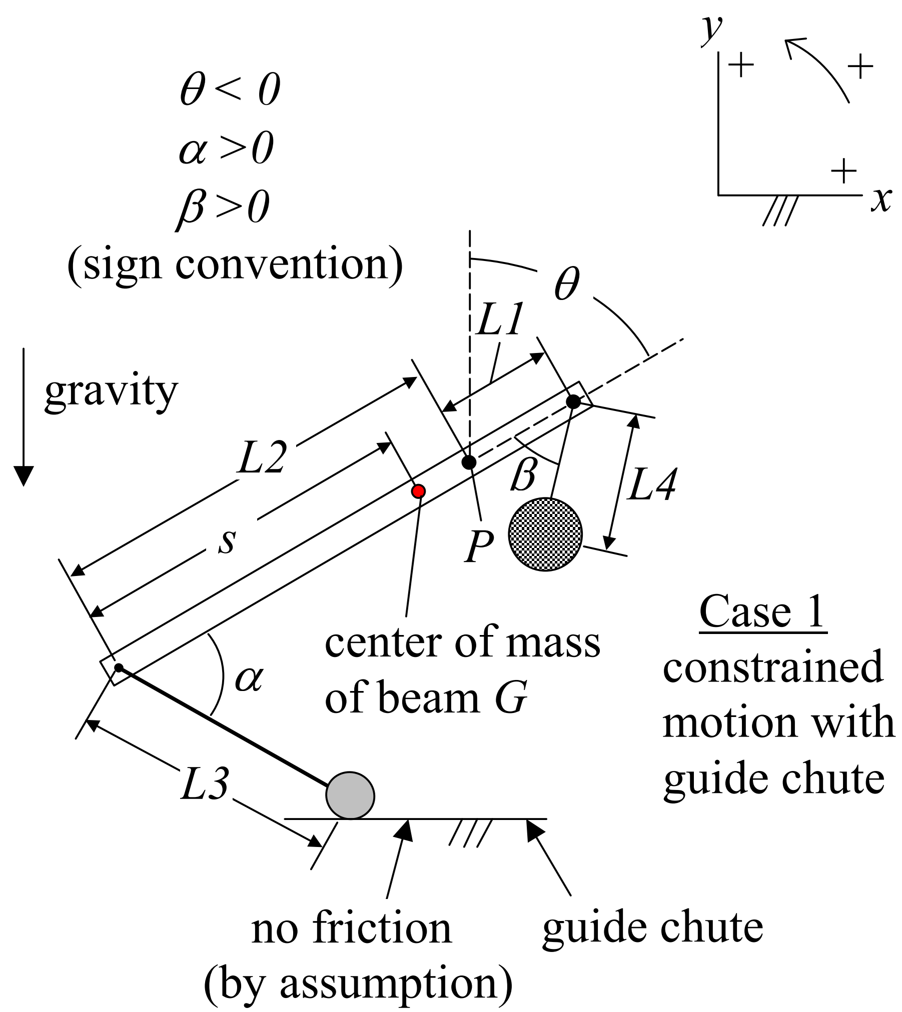



Figure 2

Note that the counterweight is modeled as a hinged object with center of mass located a distance *L4* from the end of the beam

Figure 3 below shows the unconstrained case, where the payload has lost contact with the guide chute and has "lifted off". At this point the physics of the problem changes since the payload is no longer in contact with the guide chute. The initial conditions for this case (at the instant lift off occurs) are equal to the (final) conditions from Case 1 (just before the payload loses contact with the guide chute).

If you do not wish to use a guide chute in your design, you can just set initial conditions of your choice in the Excel spreadsheet (in cells J5-J10). This corresponds to an unconstrained case (as shown in the figure below).

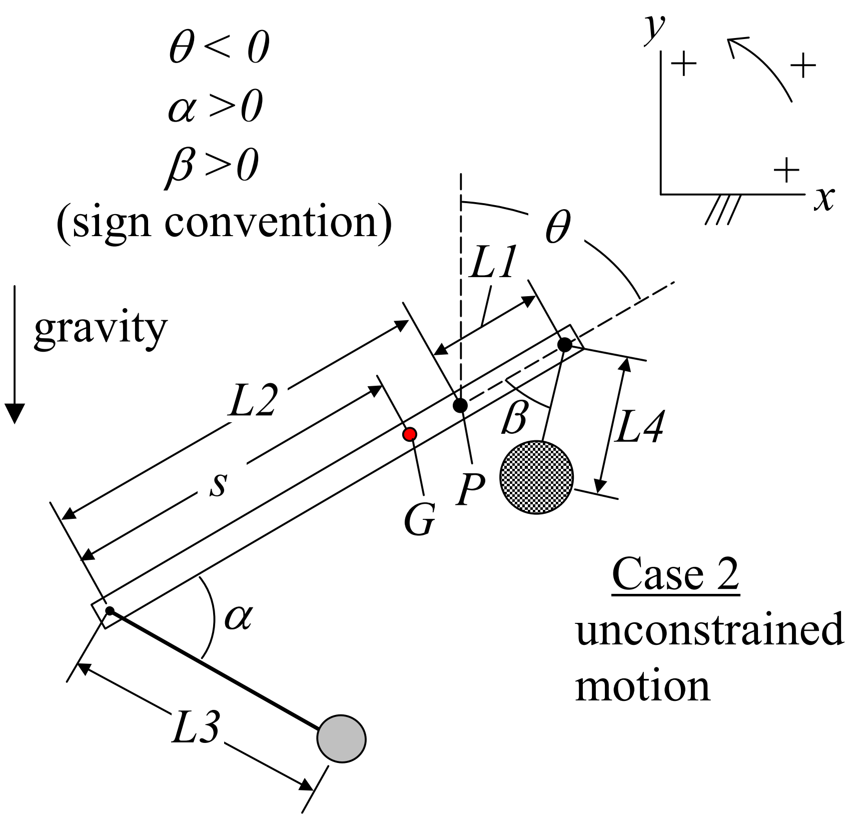
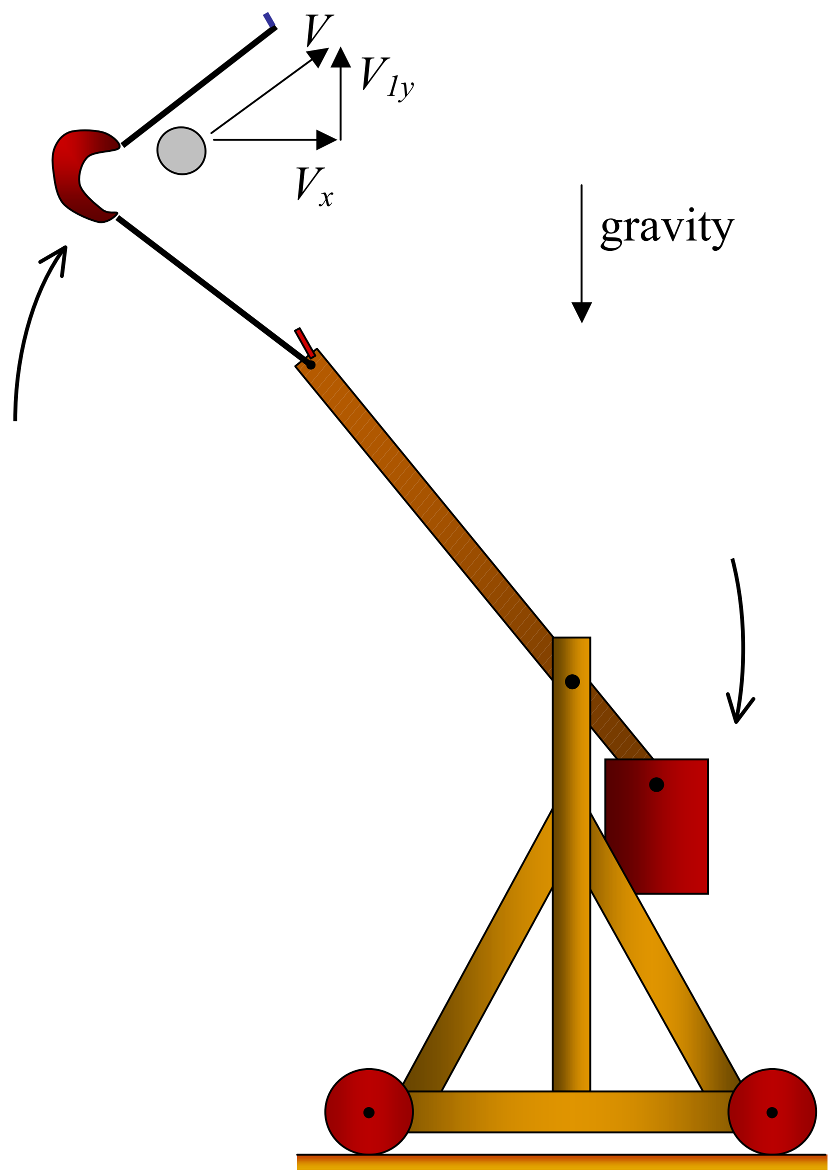
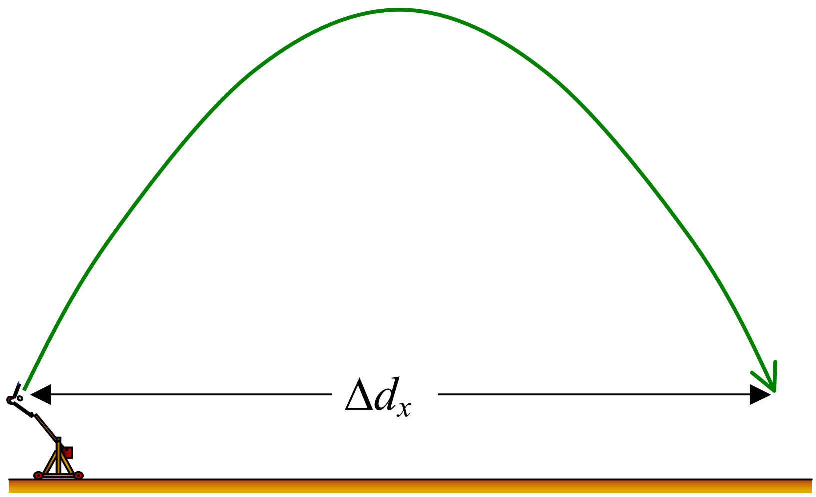



Figure 3

The figure below shows the velocity of the payload at the instant the sling releases. At this point the payload can be treated as a projectile, where the vertical component of velocity $V_y$ changes (due to gravity), and the horizontal component of velocity $V_x$ stays constant (since air resistance is neglected). The initial vertical component of velocity is given as $V_{1y}$ , shown below.



The figure below shows the trajectory of the payload after launch.




The approximate horizontal distance traveled by the payload is given by:

$$\Delta d_x = V_x t$$

where

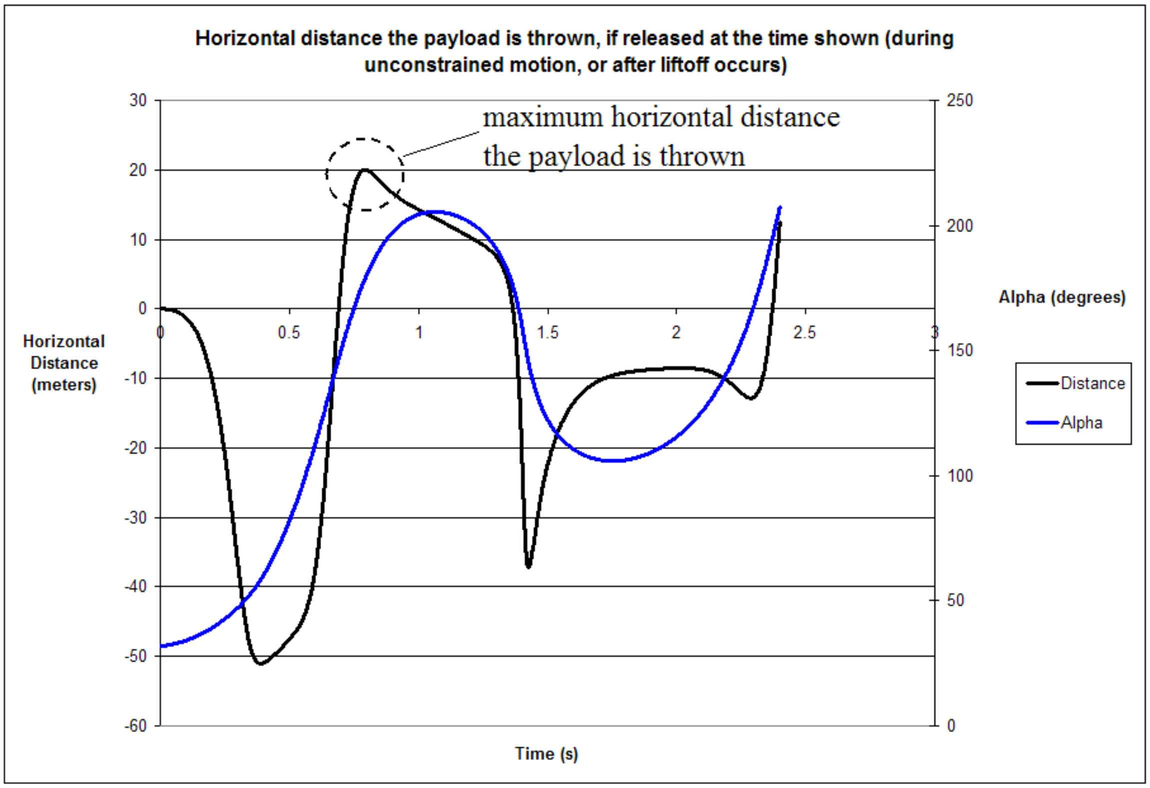
$$t = 2\frac{V_{1y}}{g}$$ ($t$ is time and $g$ = 9.8 m/s$^2$ – the acceleration due to gravity, on earth)

Therefore,

$$\Delta d_x = \frac{2V_x V_{1y}}{g}$$

In the Excel spreadsheet, an example of Chart 3 (shown below) gives the horizontal range $\Delta d_x$ the payload is thrown, as a function of release time. The maximum horizontal distance thrown (the first positive peak of the graph) corresponds to the desired sling release angle $\alpha$ (alpha). Since different trebuchet designs result in different peak horizontal distances, it is evident that one would wish to adjust the dimensions of the trebuchet (and initial release conditions) so that the peak horizontal distance thrown is maximized.

Note: Only the **first** positive peak of the graph is valid, in terms of maximum distance thrown. The sling angle $\alpha$ is continually increasing up to the first positive peak, and (as a result) it's easy to set the sling release angle here.




## Input Variables In Excel Spreadsheet

In the Excel spreadsheet, the values in the green cells are what you change. These are the input values. These input values are shown in Figures 1-3 (with the exception of $I_p$ which is described on the next page). For the most part, the input values are self-explanatory. However, there are certain input values for which it may not be clear what they represent. These values are listed below, along with their meaning:

theta – $\theta$ (this value is negative in the model, in accordance with the sign convention used. However, in the Excel charts, $\theta$ is plotted as positive, to avoid confusion)

d(theta)/dt – $d\theta/dt$

alpha – $\alpha$ (this value is positive, in accordance with the sign convention used)

d(alpha)/dt – $d\alpha/dt$

beta – $\beta$ (this value is positive, in accordance with the sign convention used)

d(beta)/dt – $d\beta/dt$

These above values are given in rads (radians) and rads/s (radians/second). If you are using degrees in your design you must convert it to radians when entering the input values in the Excel spreadsheet. One radian is equal to 57.29578 degrees. For example, theta = 45 degrees is equal to 45/57.29578 = 0.785 rads. Another example: d(theta)/dt = 105 degrees/s is equal to 1.83 rad/s.

The radian measure is a common convention used in physics and engineering. It simplifies certain types of calculations, which is why it is used in the model.

Note: If the trebuchet is released from a stationary position (i.e. it starts from rest), then d(theta)/dt = 0, d(alpha)/dt = 0, and d(beta)/dt = 0.

Note: If you are using a guide chute in your design, make sure the contents of cells S7-S12 are copied into cells J5-J10 for each set of input values. This is specified in the spreadsheet.

Note: The Excel simulation spans enough time (in seconds) to ensure that the simulation is captured in its entirety for most every trebuchet design; meaning, the desired sling release angle $\alpha$ (for maximum throwing distance) lies well before the end of the simulation is reached.

Reminder: The sign convention for the values used in the Excel spreadsheet is given in Figures 1-3. This is the sign convention you **must** follow when entering the input values in the Excel spreadsheet (these are the values in the green cells).

The moment of inertia $I_p$ of the beam is the moment of inertia of the beam about an axis passing through the fixed pivot $P$ and pointing out of the page. It is given by:

$$I_P = I_G + m_b (L2 - s)^2$$

The first term on the right ($I_G$) is the moment of inertia of the beam about its center of mass $G$ (about an axis passing through point $G$ and pointing out of the page).

The second term on the right accounts for the offset of the pivot $P$ from the center of mass $G$ (parallel axis theorem).

If we assume that the beam can be approximated as a uniform slender rod:

$$I_G = \frac{1}{12} m_b (L1 + L2)^2$$

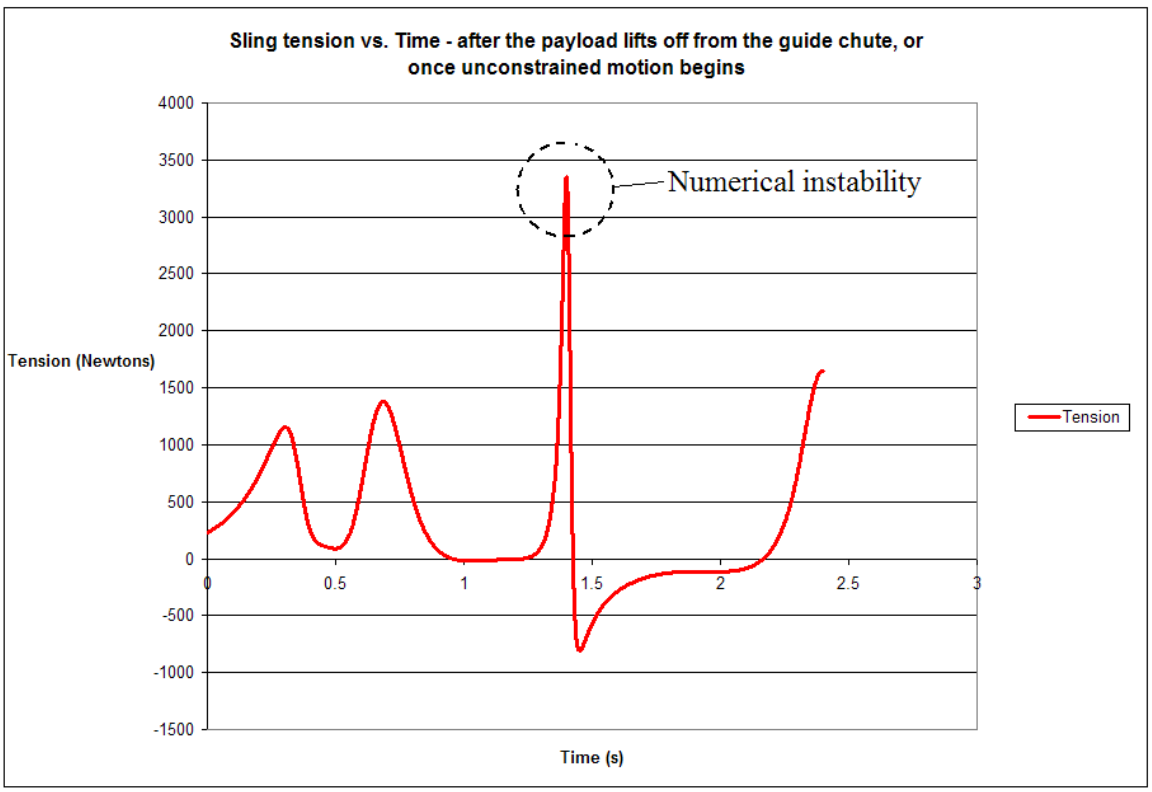
Therefore,

$$I_P = \frac{1}{12} m_b (L1 + L2)^2 + m_b (L2 - s)^2$$

The center of mass G lies in the midpoint of the beam, therefore $s = (L1 + L2)/2$. If the shape of the beam is something other than a slender rod, we must use a different value for $I_G$ and a different value for $s$.

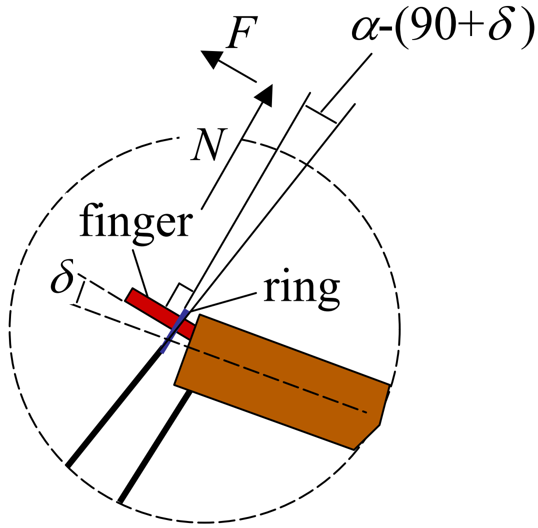
### Numerical Instability

An example of Chart 1 (shown below) shows a "spike" in the sling tension. This is an example of a numerical instability, which is unavoidable for certain choices of input parameters. It will not generally be a problem in terms of the overall solution but should be recognized as something that can happen depending on your choice of input parameters. However, the solution will only be inaccurate at the location of this instability, but accurate everywhere else.



**Friction Between Ring and Finger**

The figure below shows a schematic of the ring and finger.




The normal force $N$ acts perpendicular to the finger, due to the contact between ring and finger. The normal force is given as:

$$N = \left(\frac{T}{2}\right)\cos\left(\alpha - (90 + \delta)\right)$$

where $T$ is the sling tension in the (single cable) model as given by the Excel spreadsheet, and $T/2$ is the tension in each length of the sling (in the actual trebuchet).

The friction force $F$ acts parallel to the finger, due to the contact between ring and finger. The friction force is given as:

$$F = \left(\frac{T}{2}\right)\sin\left(\alpha - (90 + \delta)\right)$$

Let $\mu_s$ be the coefficient of static friction between the ring and finger.

When the ring starts to slip off the finger we have:

$$F > \mu_s N$$

Using the above three equations we can solve for $\delta$ when slipping begins:

$$\delta = \alpha - 180° - \tan^{-1}\left(-\frac{1}{\mu_s}\right)$$

where $\delta$, $\alpha$, and $\tan^{-1}(-1/\mu_s)$ are in degrees.

Note that the solution is independent of tension $T$, since it cancels out.

Therefore, the basic procedure when using the above equation is:

1) Determine the coefficient of static friction $\mu_s$ between the ring and finger. This can be done experimentally by setting up the finger at an arbitrary angle $\delta$ and manually increasing the sling angle $\alpha$ until the ring starts to slide off, and then from the above equation solve for $\mu_s$ (since $\delta$ and $\alpha$ are known).

2) From the Excel spreadsheet, find the sling release angle $\alpha$ corresponding to the maximum horizontal distance the payload is thrown (from Chart3). For example, from Chart3 shown on page 9, the maximum horizontal distance thrown is 20 meters, with a sling release angle $\alpha$ of around 180°.

3) Using the above equation, solve for $\delta$ using the values for $\mu_s$ and $\alpha$ calculated from 1) and 2). This value of $\delta$ for the finger angle will be used in your design. For example, if $\alpha$ = 130°, and $\mu_s$ = 0.12, then $\delta$ = 33.16°. You can use $\delta$ = 33° in your design.

It is assumed that the length of the finger is short enough (perhaps a few centimeters in length) so that when slipping begins the ring slides off the finger very quickly. However, there is a possibility that the "sliding time" could potentially delay the launching of the payload by a bit (since the sling will fully release <u>only</u> when the ring is completely off the finger). So, it may be necessary to compensate for this by making the finger angle $\delta$ a few degrees less than the value calculated from the above equation. This way, by the time the ring slides off the finger completely, the sling angle $\alpha$ will have reached the (optimal) value determined from Chart3, and the payload will be launched at the optimal point (for maximum distance).

## Optimal Design

According to Donald B. Siano (*Trebuchet Mechanics*, March 28, 2001), the optimal release position and design, based on his definition of "range efficiency" is such that:

- The initial release position is such that the beam on the counterweight side makes an angle of 45° with the vertical ($\theta_I$ = -45° in Figure 1).

- The length of the long arm of the beam is 3.75 times the length of the short arm of the beam ($L2$ = 3.75x$L1$).

- The length of the sling is equal to the length of the long arm of the beam ($L2$=$L3$).

- The length of the counterweight suspension length is such that $L1$=$L4$.

According to Siano's results, propping the counterweight up initially makes little difference. So in general, it is recommended that the counterweight hangs straight down initially, where $|\theta| = |\beta|$.

Furthermore, Siano recommends using a counterweight that has a mass 100 times greater than the mass of the payload ($M$ = 100x$m$). However, it is certainly possible to achieve a good design with a much lighter counterweight than this.

## Additional Features in Excel Program

There are other values in the Excel spreadsheet you may be interested in plotting and analyzing (which aren't already plotted in the charts), such as the reaction forces at the pivot *P* (columns AT/AU and CH/CI in the Excel spreadsheet). Note that the direction of these reaction forces follows the *x-y* sign convention given in Figures 1-3.

These reaction forces can be useful information if calculating the stresses in the beam during launch.

## ~~Troubleshooting~~

~~If the Excel spreadsheet doesn't work, or you think the results are messed up in some way, first make sure your units are consistent, and also check that your input values are realistic. But if you still can't figure out what's wrong, email me at franco@real-world-physics-problems.com. My alternate email address is: fvnorman@hotmail.com. If possible, write me with the same email address that you entered in the purchase form. This isn't necessary but it just makes it easier for me to confirm that you actually purchased the product. If you didn't purchase the product your support questions will go unanswered.~~