A snowboard half-pipe has a pitch of 18° and a wall height of 7 m. What is the optimal takeoff angle for a snowboarder to reach maximum height, maximum airborne time, and maximum distance along the half-pipe?

Solution:

$\beta = 18°$
$d = 7\,m$

By trigonometry,

$$\Delta h = \frac{d}{\cos\theta}\sin(90-\beta-\theta)$$

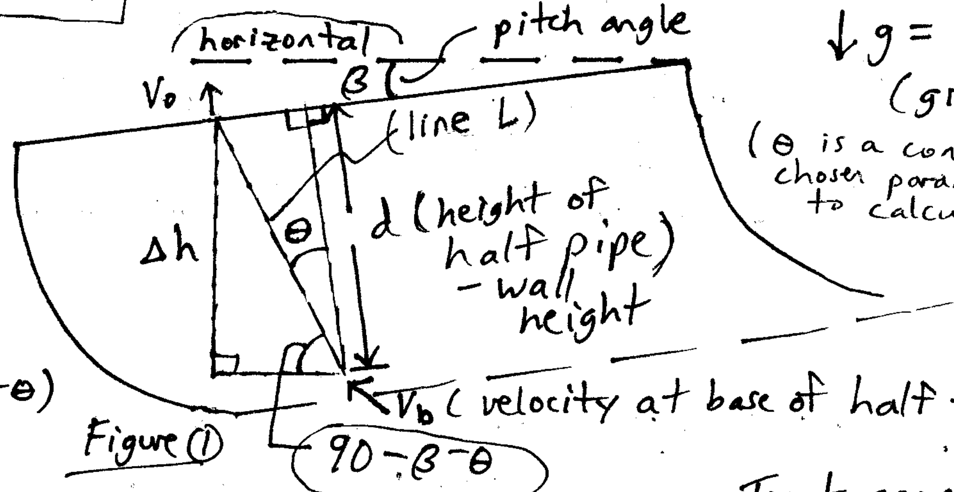


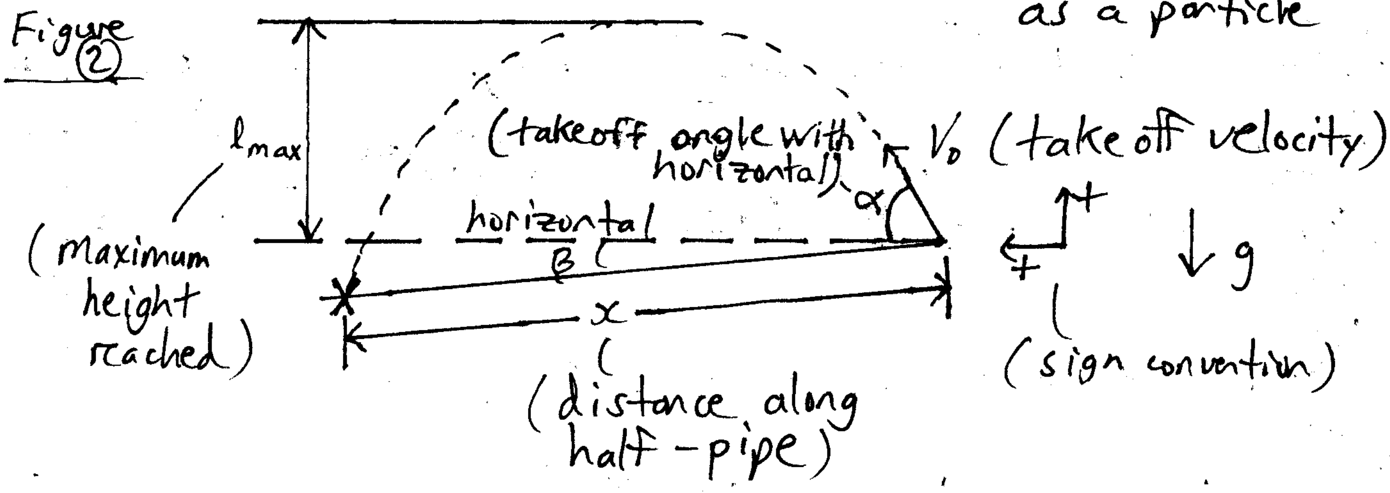
Figure (1)

$\downarrow g = 9.8\,m/s^2$ (gravity)

(θ is a conveniently chosen parameter used to calculate Δh)

Treat snowboarder as a particle

Figure (2)



First, apply the projectile motion equations, assuming no air resistance.

For horizontal motion: $x\cos\beta = (V_o\cos\alpha)t$ (1), t is time

For vertical motion: $-x\sin\beta = (V_o\sin\alpha)t - \frac{g}{2}t^2$ (2)

Combine (1) and (2) and we get:

$$x = \frac{V_o^2\cos^2\alpha}{(g/2)\cos\beta}(\tan\alpha + \tan\beta) \quad (3)$$ - distance along half-pipe

$$t = \frac{V_0 \cos\theta \,(\tan\theta + \tan\beta)}{(g/2)} \quad (4) \text{ - airborne time}$$

The maximum height reached is given by:

$$l_{max} = \frac{V_0^2 \sin^2\theta}{2g} \quad (5) \text{ - maximum height}$$

Using energy considerations we can find V_0 in terms of V_b (ignoring friction):

$$\frac{1}{2} m V_b^2 = \frac{1}{2} m V_0^2 + \underbrace{mg\Delta h}_{\text{gravitational energy}}$$

, where m is the mass of the snowboarder plus board

Thus, $V_0 = \sqrt{V_b^2 - 2g\Delta h}$

Assume the snowboarder does not "pump" for extra speed upon takeoff. So the only energy consideration is gravity.

Accounting for "pumping" would not affect the solution here since it serves to increase takeoff speed which would fall within the range of speeds considered anyway, resulting from V_b in the range 13 - 18m/s

The known variables are: β, V_b, d

We wish to maximize x, t, l_{max} in equations (3) - (5).

Now, $\beta = 18°$, $d = 7m$, and use a range for V_b of 13-18 m/s.

Typical angles of θ are in the range 0-10°.

While going up the half-pipe it is best to minimize turning since this minimizes friction and results in a greater takeoff speed V_0. Hence, the best strategy is to go straight up the half-pipe along the line L, at some angle θ. This means that $\varphi = 90 - \beta - \theta$.

Out of all the quantities to be maximized (x, t, l_{max}) the most relevant are t and l_{max}, in terms of performance criteria. Having maximum t is important because it means maximum airborne time to perform airborne turns and twists in the air. Having maximum l_{max} is also important because it means getting the most height, which is aesthetically pleasing and is a scoring criterion in some events.

Using Excel I find that maximum t is very closely achieved for $\theta = 5°$ (and $\varphi = 90 - 18 - 5 = 67°$). There is at most 0.1 seconds difference between the actual maximum t achieved, at some θ, and the value of t at $\theta = 5°$. The difference is greatest for V_b in the lower part of the range 13–18 m/s, and vanishingly small for V_b in the higher part of the range 13–18 m/s. Therefore, maximum t is achieved almost exactly at $\theta = 5°$ when $V_b = 16$–18 m/s. Maximum l_{max}, however, is not quite achieved, even closely, for $\theta = 5°$. The difference between l_{max} at $\theta = 5°$ and the maximum l_{max}, at some θ, is at most 0.5 m (50 cm). But the height reached is still high, especially for the higher speeds. For example, at $\theta = 5°$, $l_{max} = 8.4$ m, for $V_b = 18$ m/s. But at $V_b = 13$ m/s and $\theta = 5°$, $l_{max} = 1.8$ m. Clearly, higher speeds is essential for achieving big height.

In fact, for every additional 1 m/s of speed V_b in the range 13-18 m/s, we get at least 1.2 m in extra height in l_{max}. This is at least 4 feet!

Sample calculation:

For $V_b = 18$ m/s, t is maximum at $\theta = 6°$ (roughly)
and $t = 3.0$ s
and $l_{max} = 8.4$ m
$x = 18.29$ m
$V_0 = 14.08$ m/s
$\vartheta = 66°$