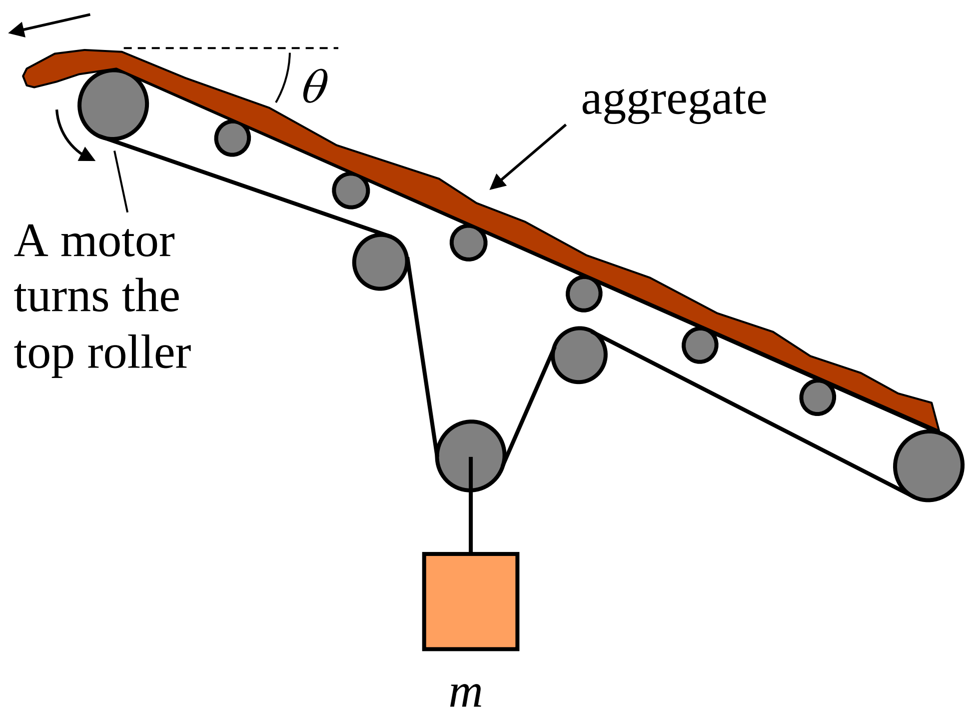
Problem

A conveyor belt carrying aggregate is illustrated in the figure below. A motor turns the top roller at a constant speed, and the remaining rollers are allowed to spin freely. The belt is inclined at an angle θ. To keep the belt in tension a weight of mass m is suspended from the belt, as shown.

Find the point of maximum tension in the belt. You don't have to calculate it, just find the location and give a reason for it.



Solution

To determine the point of maximum belt tension, make six imaginary cuts on the belt at the locations shown in the figure below (close to the rollers), and at these locations let T_1, T_2, T_3, T_4, T_5, T_6 represent the (respective) belt tensions.

$T_1 > T_6$ because the belt at location 1 has to support the cable tension T_6 plus a fraction of the belt weight between locations 1 and 6, plus a fraction of the weight of the aggregate between locations 1 and 6. Only a fraction of the weight is supported because the conveyor is inclined at angle θ. This means that only a fraction of the full force of gravity acts in the direction of the belt.

$T_2 > T_3$ because the belt at location 2 has to support the cable tension T_3 plus a fraction of the belt weight that is hanging underneath location 2, and between locations 2 and 3.

$T_3 > T_4$ because there is a longer length of belt hanging below location 3 than there is hanging below location 4.

$T_4 > T_5$ because the belt at location 4 has to support the cable tension T_5 plus a fraction of the belt weight that is hanging underneath location 4, and between locations 4 and 5.

$T_5 \cong T_6$ since locations 5 and 6 are very close to each other and the roller spins freely.

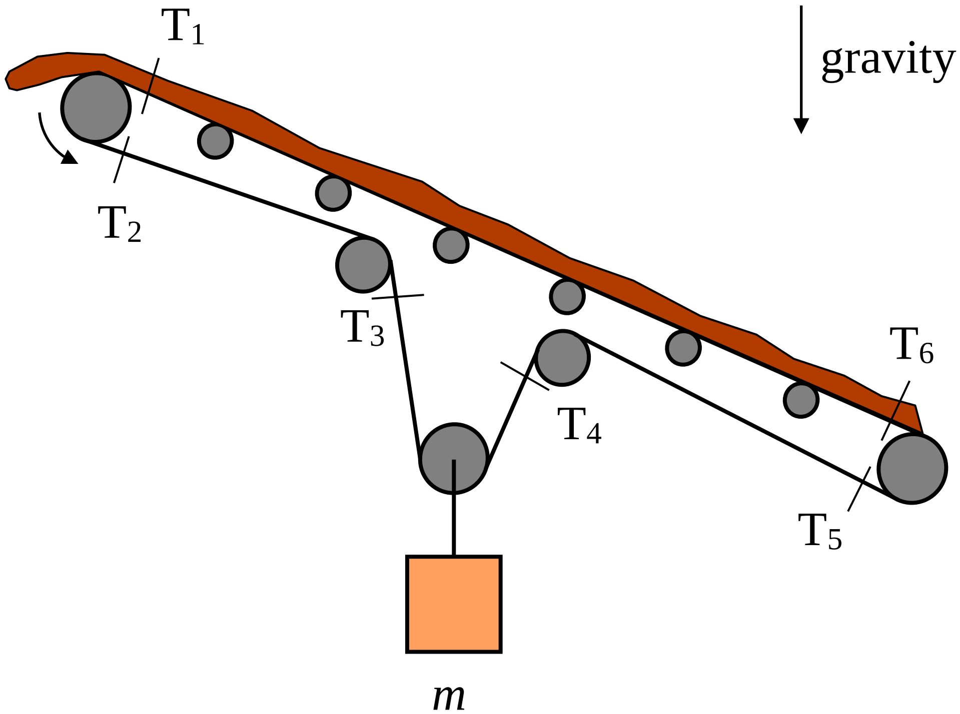
Now, the top roller is turning counter-clockwise at constant rotational speed, which means a counterclockwise motor torque is turning it in this direction. T_1 exerts a clockwise torque on the top roller, and T_2 exerts a counterclockwise torque on the top roller. The roller is turning at constant rotational speed so it is in rotational equilibrium. If we define a sign convention where counterclockwise is positive and clockwise is negative we can write:

$(\text{motor torque}) + rT_2 - rT_1 = 0$, where r is the radius of the top roller.

This gives us:

$T_1 = T_2 + (\text{motor torque})/r$

Therefore, $T_1 > T_2$ and based on the inequalities given above, the maximum belt tension is T_1.


T1
gravity
T2
T3
T6
T4
T5
m