User Manual For Projectile Motion Simulator In Excel

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The results of the program are accurate as far as the physics and mathematics are concerned. But you are still expected to exercise good judgment when using the simulation results for your modeling problems. Therefore, I am not responsible for the use or misuse of the program, or the information presented here.

Purpose

The purpose of this Excel program (spreadsheet) and manual is to allow you to model the motion of an object (projectile) under the influence of gravity, drag and (optionally) the Magnus effect (described in the next section). For example, baseballs, tennis balls, golf balls, soccer balls, and volleyballs are projectiles that experience drag and (sometimes) the Magnus effect during flight. So it is important to have a way of predicting their motion with reasonable accuracy, since accounting only for the effect of gravity can introduce significant errors in the predictions.

Analysis and Assumptions

A drag force is the resistance force caused by the motion of an object through a fluid, such as water or air. A drag force opposes the motion of an object and acts opposite to the direction of the oncoming flow velocity. This is the relative velocity between the object and the fluid. For objects of macroscopic size that are moving through the air at a speed of at least a few meters/second, the drag force (F_D) is given by:

$$F_D = \frac{1}{2} C_d \rho A v^2$$

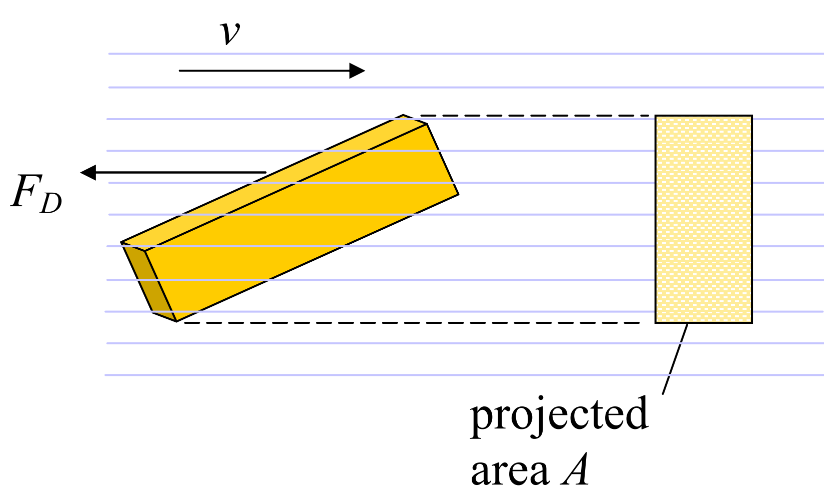
Where:

C_d is the drag coefficient, which can vary along with the speed of the object. But typical values range from 0.4 to 1.0 for different fluids (such as air and water)

ρ is the density of the fluid through which the object is moving

v is the speed of the object relative to the fluid

A is the projected cross-sectional area of the object perpendicular to the flow direction (that is, perpendicular to v). This is illustrated in the figure below, along with the direction of the velocity v and the direction of the drag force F_D.



For example, for a spherical object of radius r, $A = \pi r^2$.

If we have a spherical object (such as a ball) spinning during its motion through a fluid (such as air), friction between the object and fluid causes the fluid to react to the direction of spin of the object, generating a force known as the Magnus force. If the rotation of the object is in the plane of motion, the Magnus force (F_M) is given by:

$$F_M = K_s w \cdot v$$

Where:

w is the angular velocity of the spinning object

v is the speed of the object relative to the fluid

K_s is the proportionality constant. This constant can be expressed in a more precise mathematical form:

$$K_s = \frac{1}{2} C_s \rho A \cdot r$$

Where:

C_s is the spin coefficient, which can vary along with the speed of the spherical object. But typical values for spheres range from 0.25 to 1.0

ρ is the density of the fluid through which the spherical object is moving

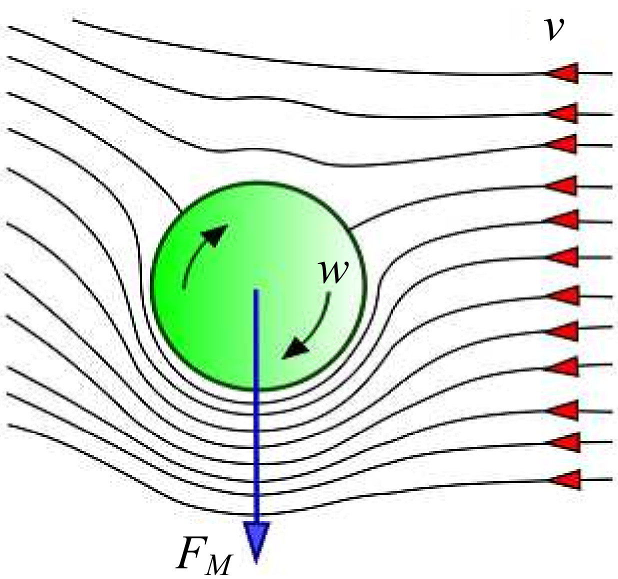
A is the cross-sectional area of the spherical object, which is equal to $A = \pi r^2$

r is the radius of the spherical object

The reference for the above equation and the range of spin coefficient is:

Optimizing A Volleyball Serve, Dan Lithio, Hope College, and Eric Webb, Case Western Reserve University, October 14, 2006.

The figure below shows a spherical object spinning in the plane of motion and acted upon by the Magnus force.



Source:

http://en.wikipedia.org/wiki/Magnus_effect
http://en.wikipedia.org/wiki/User:Gang65

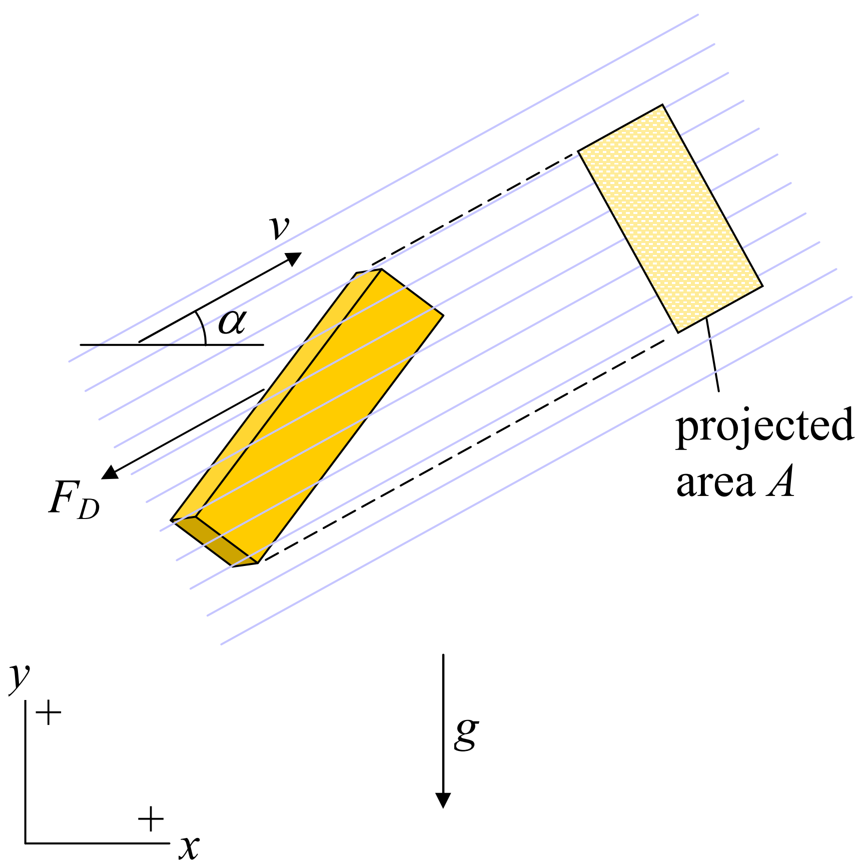
To get a better understanding of the Magnus effect, imagine the above figure is of a ball thrown through the air. As the ball spins, friction between the ball and air causes the air to react to the direction of spin of the ball.

As the ball undergoes top-spin (shown as clockwise rotation in the figure), it causes the velocity of the air around the top half of the ball to become less than the air velocity around the bottom half of the ball. This is because the tangential velocity of the ball in the top half acts in the opposite direction to the airflow, and the tangential velocity of the ball in the bottom half acts in the same direction as the airflow. In the figure shown, the airflow is in the leftward direction, relative to the ball.

Since the (resultant) air speed around the top half of the ball is less than the air speed around the bottom half of the ball, the pressure is greater on the top of the ball. This causes a net downward force F_M to act on the ball. This is due to Bernoulli's principle, which states that when air velocity decreases, air pressure increases (and vice-versa).

If the ball were to spin counterclockwise (in the opposite direction) then the situation would reverse, and the pressure on the bottom of the ball would be greater than the pressure on the top of the ball, and a net upward force F_M would act on the ball.

We will now derive the equations of motion for an object experiencing gravity and drag. Consider the general object shown in the figure below, experiencing drag, and moving at instantaneous velocity v and angle α to the horizontal.



For the object shown in the figure above, treat it as a particle, and apply Newton's second law in the x-direction:

$$-F_D \cos\alpha = ma_x$$

Where:

a_x is the acceleration of the object in the x-direction

m is the mass of the object

α is the angle between the direction of motion and the horizontal

Since,

$$a_x = \frac{dv_x}{dt}$$

we can write

$$-F_D \cos\alpha = m\frac{dv_x}{dt} \qquad (1)$$

Now, apply Newton's second law in the y-direction:

$$-F_D \sin\alpha - mg = ma_y$$

Where:

a_y is the acceleration of the object in the y-direction

g is the acceleration due to gravity

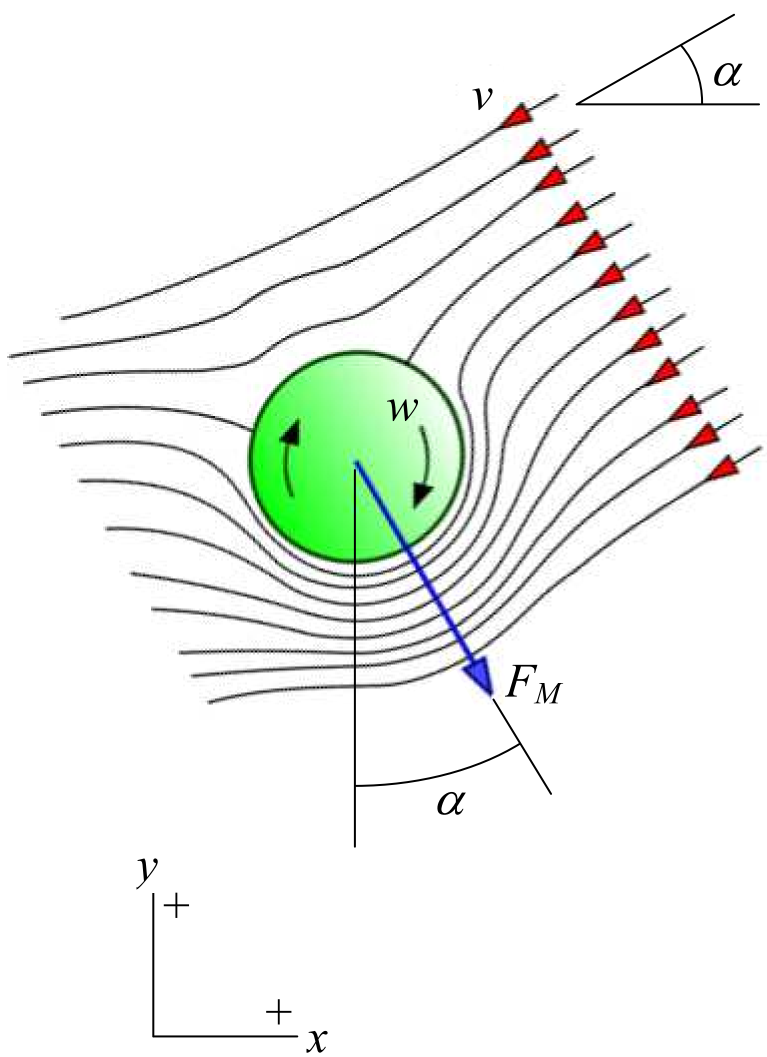
Since,

$$a_y = \frac{dv_y}{dt}$$

we can write

$$-F_D \sin\alpha - mg = m\frac{dv_y}{dt} \qquad (2)$$

Now, for the case where we have a spinning spherical object, we must add the force contribution of the Magnus effect to the force balance in equations (1) and (2). This allows us to account for both drag and the Magnus effect. Consider the spherical object shown in the figure below, spinning in the plane of motion, and experiencing the Magnus effect. It is moving at instantaneous velocity v and angle α to the horizontal.



The component of F_M in the x-direction is:

$$F_{Mx} = F_M \sin\alpha \qquad (3)$$

The component of F_M in the y-direction is:

$$F_{My} = -F_M \cos\alpha \qquad (4)$$

Next, add the Magnus force contributions of (3) and (4) to the force balance in equations (1) and (2). This gives us

$$-F_D \cos\alpha + F_M \sin\alpha = m\frac{dv_x}{dt} \qquad (5)$$

and

$$-F_D \sin\alpha - mg - F_M \cos\alpha = m\frac{dv_y}{dt} \qquad (6)$$

From before,

$$F_D = \frac{1}{2} C_d \rho A v^2$$

and

$$F_M = -K_s w \cdot v$$

Note that in the above equation we introduce a negative sign to account for the sign convention chosen for w, as explained on page 10.

From geometry,

$$\cos\alpha = \frac{v_x}{v}$$

and

$$\sin\alpha = \frac{v_y}{v}$$

and

$$v = \sqrt{v_x^2 + v_y^2}$$

Substitute the above five equations into equations (5) and (6). We get

$$-\frac{1}{2} C_d \rho A v_x \sqrt{v_x^2 + v_y^2} - K_s w \cdot v_y = m \frac{dv_x}{dt}$$

and

$$-\frac{1}{2} C_d \rho A v_y \sqrt{v_x^2 + v_y^2} - mg + K_s w \cdot v_x = m \frac{dv_y}{dt}$$

The above two equations are the final equations of motion (in the *xy* plane) for an object (projectile) experiencing the force of gravity, drag, and the Magnus effect. If the Magnus effect is not present simply set $K_s = 0$ in the above two equations, and only the force of gravity and drag will influence the motion of the object.

The following assumptions are made in this model:

- The flight of the object is in air. The equation for drag given on page 1 is most accurate for projectile motion in air, due to the negligible effects of viscous forces, which is true for objects moving at velocities of at least a few meters/second (which is almost always true for projectile motion problems).

- There is negligible wind, which can "push" on the object.

- The object does not have an airfoil shape, so lift force is negligible.

- The variables C_d, ρ, A, K_s, and w remain constant during the flight of the object. In particular for spherical objects, the projected frontal area A certainly does not change during its flight. But for non-spherical objects that tumble through the air, A changes, so an average value of A can be used.

- The variable K_s only applies for spinning spherical objects, where the Magnus effect occurs. Otherwise, set $K_s = 0$ in the Excel program. For $K_s = 0$ the object is acted upon only by gravity and drag. For $K_s \neq 0$ the object is acted upon by gravity, drag, and the Magnus effect.

- The motion of the object occurs in the vertical plane, with gravity acting downwards in this plane. In other words, the motion of the object is two-dimensional.

- For a spinning spherical object subject to the Magnus effect, the rotation of the object is in the vertical plane (i.e. the plane of flight). This means that the angular velocity vector w of the object is normal to this plane.

Note that in reality C_d is not a constant but varies as a function of object speed, flow direction, object orientation, object size, fluid density and fluid viscosity. So it must be chosen according to the conditions experienced by the object during its flight. This will usually be an approximation anyway, so it's advised that you simulate for a range of possible values of C_d to see its effect on the object (projectile) flight path. For more information on C_d see:

http://en.wikipedia.org/wiki/Drag_coefficient

http://www.grc.nasa.gov/WWW/K-12/airplane/dragco.html

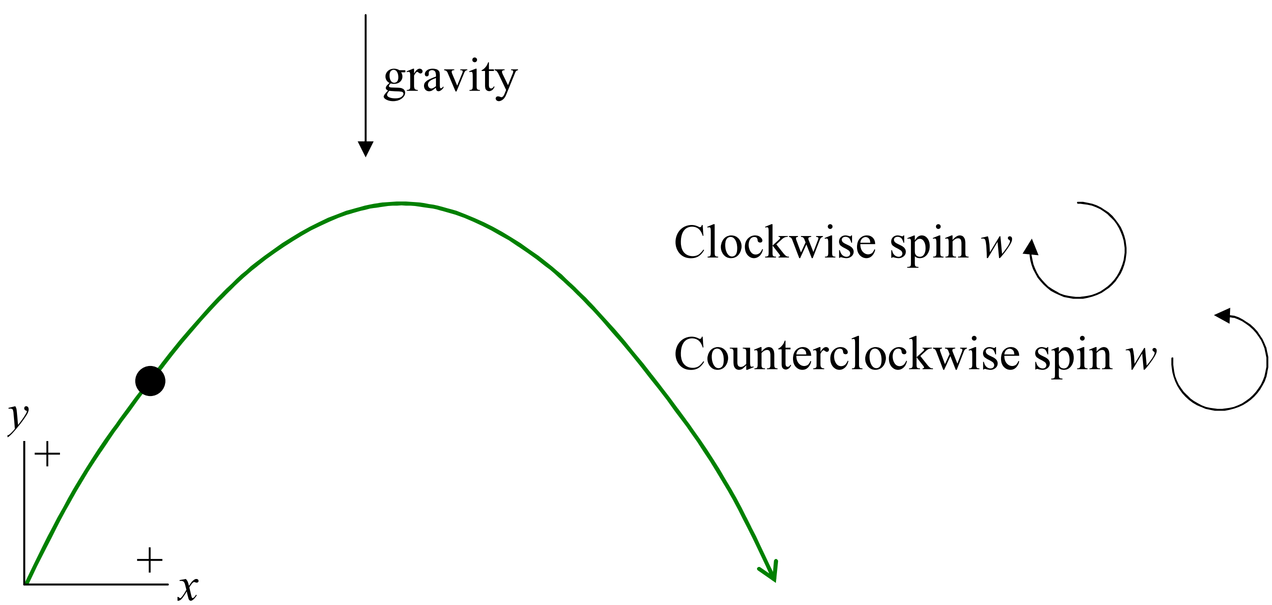
The proportionality constant K_s for the Magnus effect is also dependent on flow conditions and fluid properties. But available data on this constant is scarce, except in those cases where a sport, such as baseball, is involved, in which case the Magnus effect is important. In this instance, it is advised to use the range of spin coefficients (to calculate K_s) given on page 3, or use more exact data particular to the sport in question, such as is available online. For example, the following paper discusses the effect of spin on the flight of a baseball:

The effect of spin on the flight of a baseball, Alan Nathan, Department of Physics, University of Illinois, October 13, 2007.

Sign Convention

The figure below shows the trajectory of a typical object experiencing projectile motion, along with the sign convention used for motion along the *x* and *y* axes, and the sign convention used for the direction of spin of the object (if applicable). Keep this sign convention in mind when using the Excel simulator.

The coordinate system (*x, y*) is chosen such that the initial position of the object is at position (0, 0), which corresponds to time zero (the starting time of the simulation).



For a spherical spinning object ($K_s \neq 0$):

If w is clockwise then $w < 0$
If w is counterclockwise then $w > 0$

Input Variables In Excel Spreadsheet

In the Excel spreadsheet, the values in the green cells are what you change. These are the input values. For the most part, the input values are self-explanatory. However, for convenience sake the meaning of all these input values are given below:

Drag coefficient (Cd) – This is the coefficient C_d given in the drag force equation on page 1. This is a dimensionless number.

Proportionality constant (Ks) – This is the proportionality constant K_s given in the Magnus force equations on page 2 and 3. This number has units of kg.

Angular velocity (w) – This is the angular velocity w of the object (if it is spherical and spinning), as given in the Magnus force equation for F_M on page 2. This number has units of radians/second. Note that if you have a spin rate of X revolutions/second, the spin w in radians/second is 2πX. The sign convention for w is given on the previous page.

Density of medium (p) – This is the density ρ of the fluid through which the object is moving. This number has units of kg/m^3. The typical density of air is 1.2 kg/m^3.

Projected area (A) – This is the projected cross-sectional area A of the object perpendicular to the flow direction, as given in the drag force equation on page 1. This number has units of m^2.

Mass of object (m) – This is the mass m of the object. This number has units of kg.

Acceleration due to gravity (g) – This is the acceleration of the object due to gravity, which on earth is equal to g = 9.8 m/s^2. This number has units of m/s^2. If you want to take into account the buoyant force acting on the object, you can use an "effective" acceleration due to gravity to account for the buoyant force. The formula for this is:

$$g_{eff} = g - \frac{\rho V g}{m}$$

where g_{eff} is the effective gravity, ρ is the density of the fluid through which the object is moving (in kg/m^3), V is the volume of the object (in m^3), g = 9.8 m/s^2, and m is the mass of the object (in kg). Note that the value for g_{eff} goes into cell H10 in the Excel spreadsheet.

Note that buoyant force is often negligible but can sometimes be significant. For example, the buoyant force acting on a basketball is significant because its volume mostly consists of pressurized air, which weighs little – so it results in an effective acceleration due to gravity of $g_{eff} = 9.66$ m/s^2.

Initial Vx – This is the initial *x*-velocity of the object at starting time zero. This corresponds to the initial *x*-velocity of the object at (*x*, *y*) coordinate (0, 0). Note that if the object is launched at an initial velocity V at an angle θ above the horizontal, then Vx = Vcosθ. This number has units of m/s. The sign convention for Vx is given on page 10.

Initial Vy – This is the initial *y*-velocity of the object at starting time zero. This corresponds to the initial *y*-velocity of the object at (*x*, *y*) coordinate (0, 0). Note that if the object is launched at an initial velocity V at an angle θ above the horizontal, then Vy = Vsinθ. This number has units of m/s. The sign convention for Vy is given on page 10.

Note: The Excel simulation spans about 10 seconds, which is enough time to capture most projectile motion simulations in their entirety. The simulation time is given in column F in the spreadsheet, in seconds.

If you wish to have a simulation time longer than 10 seconds, simply copy the cells F10048-T10048 down in the spreadsheet to span enough simulation time that you need. After doing this you have to increase the ranges in the charts to span the additional simulation time, otherwise the simulation time shown on the graphs will only go up to 10 seconds. You can increase the ranges by right-clicking on the charts and then selecting Source Data, and then Series. Here you can increase the ranges. Note that these steps might vary somewhat depending on what version of Excel you are using.

Note however that since forward integration is used in the solution, you may need to reduce the time step in cell H13 to ensure accurate results. To check accuracy you can make the time step smaller in cell H13 and see if the curves in Charts 1 and 2 change significantly. If not, then your time step is small enough. But note that decreasing the time step will result in a shorter simulation time. For example, making your time step half as small results in a simulation time half as long, which means you might have to copy the cells F10048-T10048 down further in the spreadsheet to span the simulation time that you need.

Charts in the Excel Spreadsheet

Chart 1 shows the *x* versus *y* position of the object for: (1) The case of gravity+drag+magnus effect (if applicable), and (2) The case where only gravity is acting on the object. Chart 2 shows the *x* and *y* position of the object versus time, for the case of gravity+drag+magnus effect (if applicable).

In Chart 1, the initial position and the initial *x* and *y* velocity of the object is the same for both cases (1) and (2). This allows direct comparison between an object under the influence of gravity, air drag, and (optionally) the Magnus effect, and the same object with only gravity acting on it.

There are other values in the Excel spreadsheet you may be interested in plotting and analyzing (which aren't already plotted in the charts), such as the velocity of the object when there is gravity+drag (but no Magnus force) acting on the object, in order to determine terminal (constant) speed. Note that this velocity is given in columns J/K in the Excel spreadsheet. Terminal speed occurs when the object is in equilibrium, in which the drag force balances out the force of gravity. Note that you can estimate the time it takes for the object to reach terminal speed by determining the time it takes for the object to reach 99% of its (final) terminal speed.

In the charts, the two axes scales are set automatically, but you can manually change the scale in the axes to what you want. You can do this by right-clicking the axes and selecting Format Axis, and then Scale. Here you can change the axes scale.

Final Note on Magnus Effect

As mentioned previously, $K_s \neq 0$ applies only for spherical spinning objects. However, the Magnus effect also occurs in spinning cylinders subjected to fluid cross-flow. But it is highly unlikely that you will ever have to model the projectile motion of a cylinder thrown such that it spins with angular velocity vector normal to the plane of motion. Such motion would likely be unstable anyway, resulting in a tumbling motion of the cylinder as it flies through space. This tumbling motion would affect the dynamics of the problem such that the cylinder motion would likely become three-dimensional. As a result, the model used here would no longer apply. For this reason, the Magnus force in this model is only suited for spherical objects spinning in the plane of motion, which (because of their spherical shape) remain relatively stable during flight, and as a result experience two-dimensional motion in the vertical plane.

Troubleshooting

If the Excel spreadsheet doesn't work, or you think the results are messed up in some way, first make sure your units are consistent, and also check that your input values are realistic. ~~But if you still can't figure out what's wrong, email me at franco@real-world-physics-problems.com. My alternate email address is: fvnorman@hotmail.com. If possible, write me with the same email address that you entered in the purchase form. This isn't necessary but it just makes it easier for me to confirm that you actually purchased the product. If you didn't purchase the product your support questions will go unanswered.~~