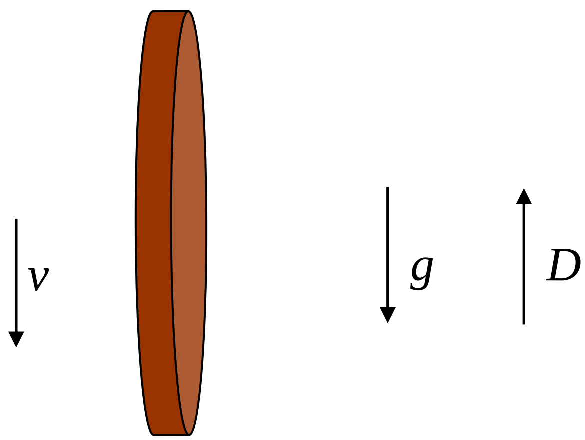
Will a penny dropped from the top of the Empire State Building kill a person or penetrate the ground?

The fastest speed a penny can achieve is terminal speed. This is the speed at which the drag force from air resistance exactly balances the force of gravity pulling down on the penny. As the penny falls it accelerates until its speed is high enough so that the corresponding air drag force exactly balances the force of gravity pulling down on the penny.

Consider the schematic showing a penny falling with the thin side facing down. This orientation results in the greatest terminal speed, as will be explained.



The general equation for the drag force acting on a body is:

$$D = \frac{1}{2} C \rho A v^2$$

Where:

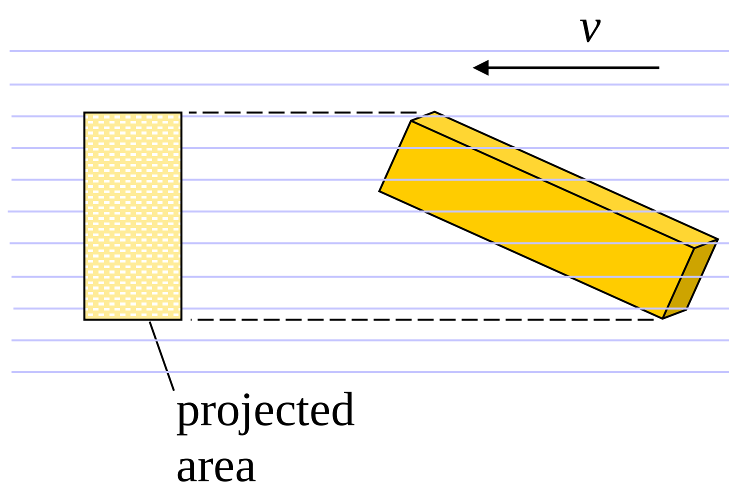
D is the drag force acting on the body

C is the drag coefficient, which can vary along with the speed of the body. But typical values range from 0.4 to 1.0 for different fluids (such as air and water)

ρ is the density of the fluid through which the body is moving (in this case, the fluid is air)

v is the speed of the body relative to the fluid

A is the projected cross-sectional area of the body perpendicular to the flow direction (that is, perpendicular to *v*). This is illustrated in the figure below.



The force of gravity pulling down on the penny is:

$$W = mg$$

Where:

W is the force of gravity pulling down on the penny

m is the mass of the penny

g is the acceleration due to gravity, which is 9.8 m/s^2

When terminal speed is reached *D* = *W* so we have

$$mg = \frac{1}{2} C \rho A v^2$$

Set $v = v_t$ and solving for terminal speed we have

$$v_t = \sqrt{\frac{2mg}{C \rho A}}$$

An important observation is that, the smaller the cross-sectional area A, the higher the terminal speed. The minimal value of A occurs when the penny is falling with the thin side facing down. In reality the penny will likely tumble through the air but in the interest of testing the validity of this myth we shall assume a "best case" scenario in which the penny is falling at the fastest possible speed.

We have the following values for a U.S. penny:

$m = 0.0025$ kg

diameter = 0.019 m

thickness = 0.0015 m

A = diameter x thickness = 0.019 x 0.0015 = 2.85×10^{-5} m^2

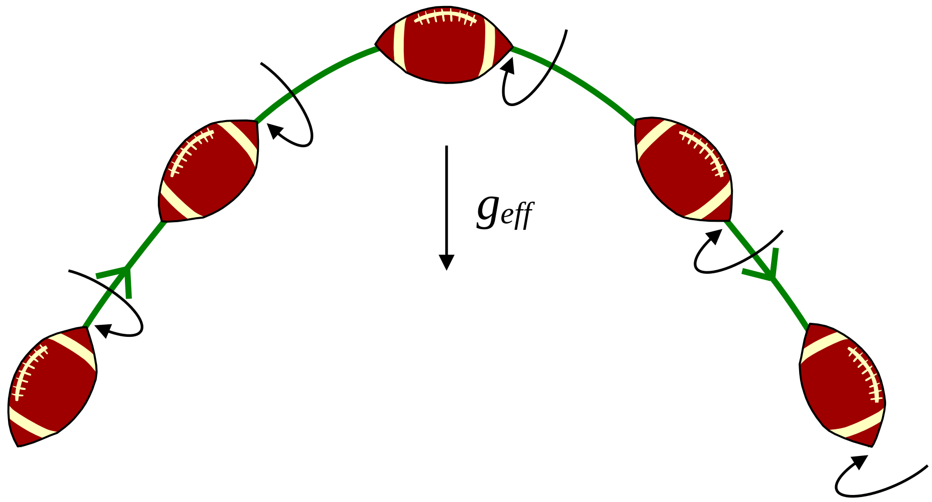
$\rho = 1.2$ kg/m^3 (density of air)

$C = 0.5$ (crude approximation based on drag coefficient for sphere as shown on http://en.wikipedia.org/wiki/Drag_coefficient)

Using the above values we get $v_t = 54$ m/s which is 190 km/h. Although high, this would not be enough speed to kill a person, or even badly injure them. But it would definitely hurt! It would also not be enough speed to penetrate a concrete surface. And keep in mind that this is the highest possible speed. The penny, as it tumbles through the air, spends some time in different orientations which produce a greater value of A, thus resulting in a lower terminal speed than the one calculated here.

Can a football fly farther if it is filled with helium?

When a football is thrown it is given spin about its axis. This creates gyroscopic stability which enables the football to keep its symmetric (long) axis aligned with its flight trajectory, without tumbling end over end when in flight. The spin imparts a gyroscopic response to the aerodynamic forces acting on the football, which results in the football long axis aligning itself with the flight trajectory (as shown below). The physics necessary to describe this is a combination of gyroscopic analysis and aerodynamic force analysis due to drag and (potentially) the Magnus effect. This is quite complicated and will not be discussed here. However, there is a lot of literature available online on gyroscope physics, as related to projectile spin and gyroscopic stability, if one wishes to study this topic further.



The fact that the football keeps its long axis aligned with its flight trajectory helps make this problem more solvable. This is because the drag force equation, used in the solution, has a constant projected frontal area as well as having a reasonably constant drag coefficient, as a result. This is directly a result of the alignment of the long axis of the football with its flight trajectory.

To start off, consider the general equation for the drag force acting on a body:

$$D = \frac{1}{2} C \rho A v^2$$

Where:

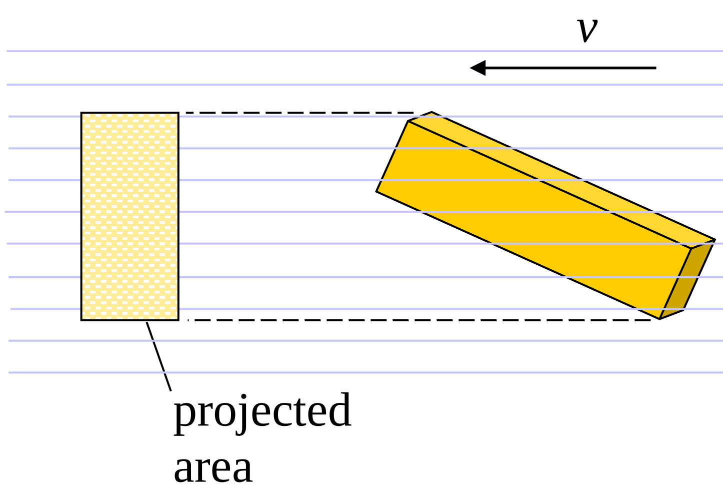
D is the drag force acting on the body

C is the drag coefficient, which can vary along with the speed of the body. For a football this value is about 0.05 (reference: http://users.df.uba.ar/sgil/physics_paper_doc/papers_phys/fluids/drag_football.pdf)

ρ is the density of the fluid through which the body is moving. In this case, the fluid is air so ρ = 1.2 kg/m^3

v is the speed of the body relative to the fluid

A is the projected cross-sectional area of the body perpendicular to the flow direction (that is, perpendicular to *v*). This is illustrated in the example figure below. For a football in flight $A = \pi r^2$, where r = 0.085 m is the radius of the football at the mid point. So A = 0.023 m^2.



The above variables stay the same whether the football is filled with air or helium.

The two variables which depend on whether the football is filled with air or helium are mass and effective gravity. In both cases the football is filled to the same pressure.

The mass of the football will vary depending on if it's filled with air or helium, since they have different densities (helium has a lower density). If a football is filled with air it will typically have a mass of about 410 grams. If it is filled with helium it will weigh about 7 grams less (reference: http://www.discovery.com/tv-shows/mythbusters/mythbusters-database/football-helium-fly-farther). So it would have a mass of 403 grams.

The effective gravity takes into account the buoyant force acting on an object. Air exerts a buoyant force on objects but its effect is usually negligible in projectile motion calculations. To calculate effective gravity we need to know the volume of a football, which is 0.0042 m^3 (reference: http://www.csus.edu/indiv/o/oldenburgj/ENGR1A/NFLFootballWtCalc.pdf).

The equation for calculating effective gravity is

$$g_{eff} = g - \frac{\rho V g}{m}$$

Where:

g_{eff} is the effective gravity

g is the acceleration due to gravity, which is 9.8 m/s^2

V is the volume of the football, which is 0.0042 m^3

ρ is the density of air which is 1.2 kg/m^3

m is the mass of the football (0.41 kg for an air filled football and 0.403 kg for a helium filled football)

For an air filled football the effective gravity is g_{eff} = 9.68 m/s^2. For a helium filled football the effective gravity is g_{eff} = 9.68 m/s^2. There is negligible difference.

Since this is a projectile motion problem we need to know the initial velocity of the football in the horizontal and vertical direction. A football in professional competition is typically thrown at about 27 m/s. It can also be thrown at various launch angles. So for the sake of argument let's test out two launch angles, say 20° and 45° above the horizontal. For the 20° launch angle the horizontal velocity is 27cos20 = 25.4 m/s, and the vertical velocity is 27sin20 = 9.2 m/s. For the 45° launch angle the horizontal velocity is 27cos45 = 19.1 m/s, and the vertical velocity is 27sin45 = 19.1 m/s.

Lastly, we shall ignore the Magnus effect due to the spinning of the ball. This is likely an unimportant effect anyway since the equation for Magnus force is independent of whether the football is filled with air or helium.

To solve this problem we need to use a suitable projectile motion simulator program, such as the one described on http://www.real-world-physics-problems.com/projectile-motion-simulator.html.

Inputting the above values into the simulator we find that an air filled football flies a (horizontal) distance of 45.8 m when launched at 20°, and a distance of 68.7 m when launched at 45°. A helium filled football flies a distance of 45.7 m when launched at 20°, and a distance of 68.6 m when launched at 45°. The difference is clearly negligible, so it makes no difference whether the football is filled with air or helium. This is what the Mythbusters concluded.

Can a bullet fired straight up in the air kill someone on the way back down?

No. On the way back down the maximum speed of the bullet will be terminal speed (due to air resistance) which is much less than the speed at which the bullet was fired. However, if the bullet was fired in a vacuum then it would fall back to earth at the same speed as it was fired, and it could indeed kill someone.

The bullet can be fired at a speed much faster than the terminal falling speed because of the explosive force of the gunpowder propelling the bullet. But once the bullet leaves the gun the only forces acting on it are gravity and air resistance. On the way down these are the only forces acting on the bullet and they combine to produce terminal speed, similar to how a skydiver reaches terminal speed on the way down.

If a person jumps out of an airplane with the last parachute, can another person jump out later and catch the person?

If the person with the parachute is spread eagled the other person can orient themselves so that their body is vertical, making themselves as streamlined as possible. This will make their aerodynamic drag force less than that of the parachute person and they will be able to catch up. Now if the parachute person attempts to minimize their aerodynamic drag by also orienting their body in the vertical position, the other person can still catch up because the parachute pack on the back of the person creates additional drag which the non-parachute person does not experience. So he can still catch up.

Does a 4,000 foot fall take 90 seconds?

According to http://mythbustersresults.com/episode94:

"The Build Team dropped a dummy from a plane at a height of 4,000 feet (1,200 m) and measured the amount of time it took for it to hit the ground. They timed the total free fall time at just 31 seconds, which would make the ninety second free fall scene in the movie impossible."

From the skydiving physics page the terminal speed is:

$$v_t = \sqrt{\frac{2mg}{C\rho A}}$$

It is known that terminal speed for a skydiver in the spread-eagle position is about 120 mph, or 54 m/s. Setting v_t = 54 m/s, m = 80 kg, g = 9.8 m/s^2 we can solve for $C\rho A$ = 0.54, from the above equation. This will be a constant term in the equations of motion used in the projectile motion simulator described in http://www.real-world-physics-problems.com/projectile-motion-simulator.html. We can use arbitrary values for C, ρ, A as long as $C\rho A$ = 0.54. For convenience use C = 0.5, ρ = 1.2 kg/m^3, and A = 0.9 m^2. Now, the plane is moving as the dummy is dropped but let's assume that the initial horizontal velocity of the dummy is zero. And of course the initial vertical velocity is also zero. Using the simulator we can now solve for how long it takes the dummy to fall 4000 feet (1200 m). We find that the falling time is 26 seconds. This is within range of the 31 seconds determined by the Mythbusters.

Will a car dropped from 4,000 feet fall faster than a speeding car?

Let's assume the car is falling nose first, which would correspond to its fastest falling speed. In this orientation we can use the standard drag coefficient for cars. Use $C = 0.25$ which is for a Toyota Prius. Use a Toyota Prius for this sample calculation, which will serve as a representative case.

The general equation for the drag force acting on a body is:

$$D = \frac{1}{2} C \rho A v^2$$

Where:

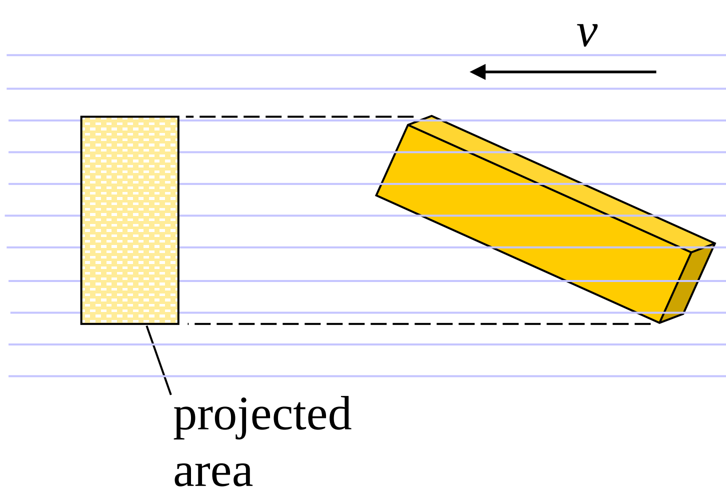
D is the drag force acting on the body

C is the drag coefficient

ρ is the density of the fluid through which the body is moving (in this case, the fluid is air where $\rho = 1.2$ kg/m^3)

v is the speed of the body relative to the fluid

A is the projected cross-sectional area of the body perpendicular to the flow direction (that is, perpendicular to *v*). This is illustrated in the figure below.



For a Toyota Prius, $A = 2.3$ m^2.

The force of gravity pulling down on the car is:

$$W = mg$$

Where:

W is the force of gravity pulling down on the car

m is the mass of the car, which for a Toyota Prius is 1300 kg

g is the acceleration due to gravity, which is 9.8 m/s^2

When terminal speed is reached $D = W$ so we have

$$mg = \frac{1}{2} C\rho A v^2$$

Set $v = v_t$ and solving for terminal speed we have

$$v_t = \sqrt{\frac{2mg}{C\rho A}}$$

For a Toyota Prius we have v_t = 192 m/s, which is 430 mph. This is much faster than a speeding car. But this assumes that the car reaches terminal speed by the time it reaches the ground. This is actually not the case. According to the results of the projectile motion simulator as described in http://www.real-world-physics-problems.com/projectile-motion-simulator.html, the car reaches a speed of 130 m/s just before it hits the ground, which is equal to 290 mph, which is still faster than a speeding car.

However this still assumes an ideal situation where the car is pointed nose down for the entirety of the fall. In reality it will experience somewhat chaotic motion as it falls through the air, which affects the aerodynamic drag and as a result will likely affect the maximum speed reached by a significant amount. However, from this result we can conclude that the car will reach a speed close to that of the fastest cars out there.