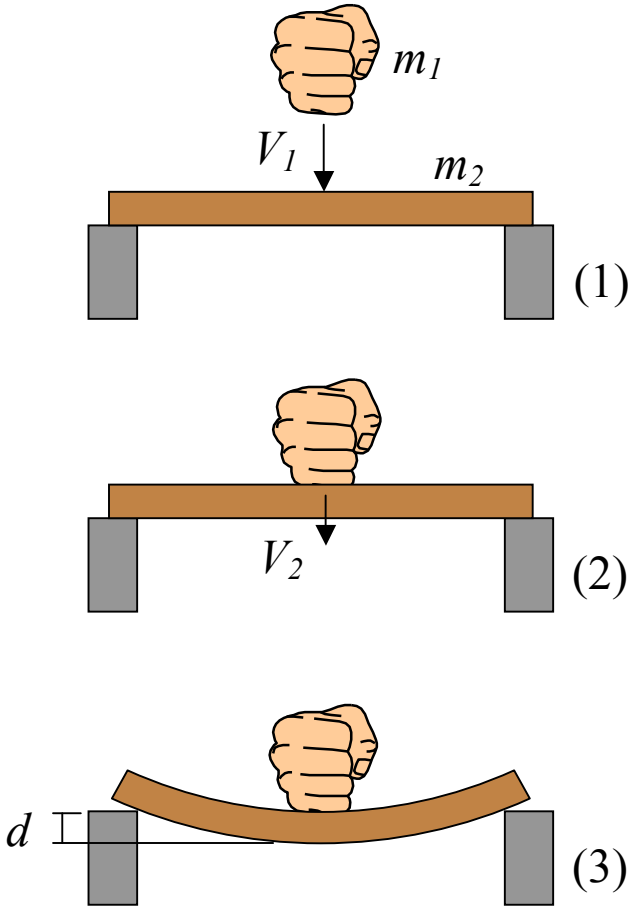
Next Generation Science Standards

HS-PS2-2: Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.

Example Problem - Karate strike

During a karate strike, a fist strikes a board at speed V_1. The fist and board experience a completely inelastic collision and the board begins to bend immediately after the collision. Immediately after the collision, the fist and board both have a speed V_2. The board breaks when it has deflected a distance d = 15 mm. The spring constant for the board is 4.1 x 10^4 N/m. The mass of the fist is m_1 = 0.65 kg and the mass of the board is m_2 = 0.15 kg.

Find the minimum initial fist speed V_1 required to break the board.



Solution:

Apply conservation of linear momentum between stage (1) and (2). We have

$$m_1 V_1 + m_2 (0) = m_1 V_2 + m_2 V_2$$

The above equation becomes

$$m_1 V_1 = (m_1 + m_2) V_2 \quad (1)$$

Immediately after the collision, the kinetic energy of the fist + board system is given by

$$KE = \frac{1}{2}(m_1 + m_2) V_2^2 \quad (2)$$

For the minimal fist speed condition this kinetic energy is equal to the energy of the board when it breaks, at which point the speed of the fist and board is zero. This means that the initial fist speed is just high enough to bend the board to the point of breaking.

The energy stored in the board at the breaking point is equal to the energy of an analogous spring with the same spring constant. Hence this spring energy is equal to

$$SE = \frac{1}{2} \times 4.1 \times 10^4 \times d^2 = 4.6 \text{ J} \quad (3)$$

At the breaking point $KE = SE$. Combine equations (1)-(3) to solve for V_1. We get V_1 = 4.2 m/s. This is the minimum initial fist speed.