Next Generation Science Standards

HS-PS3-2: Develop and use models to illustrate that energy at the macroscopic scale can be accounted for as a combination of energy associated with the motions of particles (objects) and energy associated with the relative position of particles (objects).

Example Problem - Maximum falling distance in bungee jump

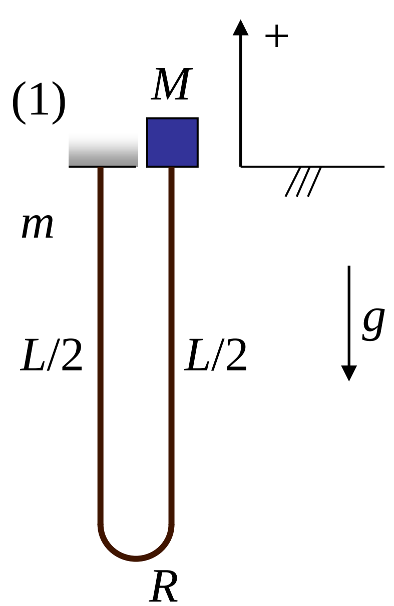
The following schematic for this analysis shows a simplified representation of a bungee jumper and bungee cord, at the initial position (1), before he jumps. The jumper is represented as a point object of mass *M*. The bungee cord is represented by two lengths of rope, each with length *L*/2, with a bend at the bottom of radius *R*. The left side of the bungee cord is attached to a fixed support. The mass of the bungee cord is *m*. The acceleration due to gravity is *g* (equal to 9.8 m/s^2 on earth). The sign convention is "up" as positive and "down" as negative.

Additional assumptions in this analysis are:

• Friction and air resistance can be neglected.

• The radius *R* of the bend is small relative to the length of the straight sections of the bungee cord. Thus, the length of the bungee cord is approximately *L*.

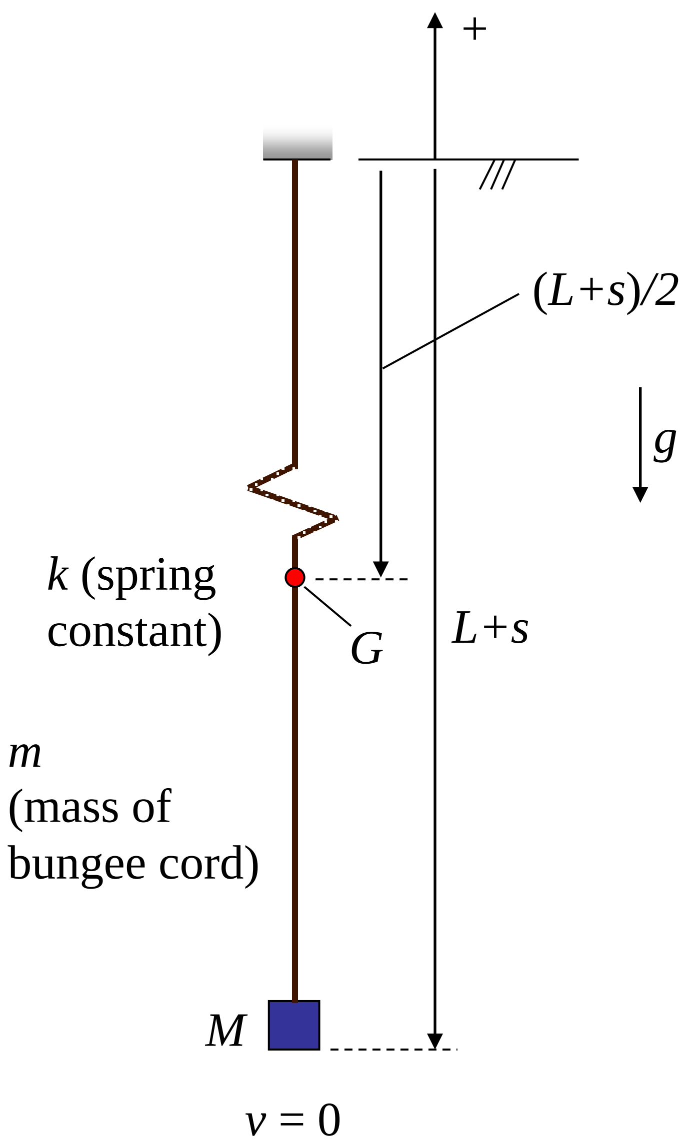
The position of the jumper is set as a function of *y* which is the position of the jumper *M* relative to the datum (chosen as his original vertical position).



Once the bungee jumper falls a distance $y = -L$, the bungee cord loses slack and tightens. This forceful transition is a form of [inelastic collision] between bungee jumper and cord, and must therefore be accounted for in this analysis in order to make accurate predictions. To solve for the velocity of the jumper immediately *after* the cord pulls tight, one needs to experimentally determine how much energy is lost during the "collision". Once this new velocity is calculated, the conservation of energy can once more be applied in order to determine the maximum falling distance of the jumper.

However, for illustrative purposes this energy loss will be ignored, and we shall apply conservation of energy to determine how far the bungee jumper falls, based on his initial position before jumping.

This analysis is set up using the schematic shown below, representing the lowest position in the fall, corresponding to the maximum stretch length.



Where:

L is the (unstretched) length of the bungee cord

s is the amount the bungee cord has stretched

G is the center of mass of the bungee cord

k is the spring constant of the bungee cord, which is assumed to behave as a linear elastic spring

The maximum distance the bungee jumper falls corresponds to the lowest point in the fall, where the velocity of the system is zero ($v = 0$). This means that the kinetic energy of the system at the lowest point is zero.

Thus, we can set up the conservation of energy equations for the system, where position (1) corresponds to the initial (rest) position and position (2) corresponds to the lowest point in the fall (where kinetic energy is zero).

Now,

$$T_1 + V_1 = T_2 + V_2 \qquad (1)$$

Where:

T_1 is the kinetic energy of the bungee jumper and bungee cord, at position (1)

V_1 is the gravitational potential energy of the bungee jumper and bungee cord, at position (1)

T_2 is the kinetic energy of the bungee jumper and bungee cord, at position (2)

V_2 is the gravitational and elastic potential energy of the bungee jumper and bungee cord, at position (2)

Now,

$$T_1 = 0 \qquad (2)$$

$$V_1 = -\frac{mgL}{4} \qquad (3)$$

Note that the gravitational potential energy of the bungee cord corresponds to the gravitational potential energy of its center of mass. At position (1) its center of mass is located at position $y = -L/4$. Therefore its gravitational potential energy at position (1) is $mg(-L/4) = -mgL/4$.

Now,

$$T_2 = 0 \qquad (4)$$

$$V_2 = -mg\left(\frac{L+s}{2}\right) - Mg(L+s) + \frac{1}{2}ks^2 \qquad (5)$$

Note that the last term in the above equation represents the elastic potential energy of the bungee cord, which is assumed to behave as a linear elastic spring.

Substitute equations (2)-(5) into equation (1). You can then solve for s numerically or with the quadratic roots formula.

For example, if k = 500 N/m, L = 15 m, g = 9.8 m/s^2, and $m = M$ = 60 kg, the amount of stretch of the bungee cord is s = 8.6 m. Now, since some energy is actually lost when the bungee cord loses slack and pulls tight, this amount of stretch is a bit higher than it would be in real life.