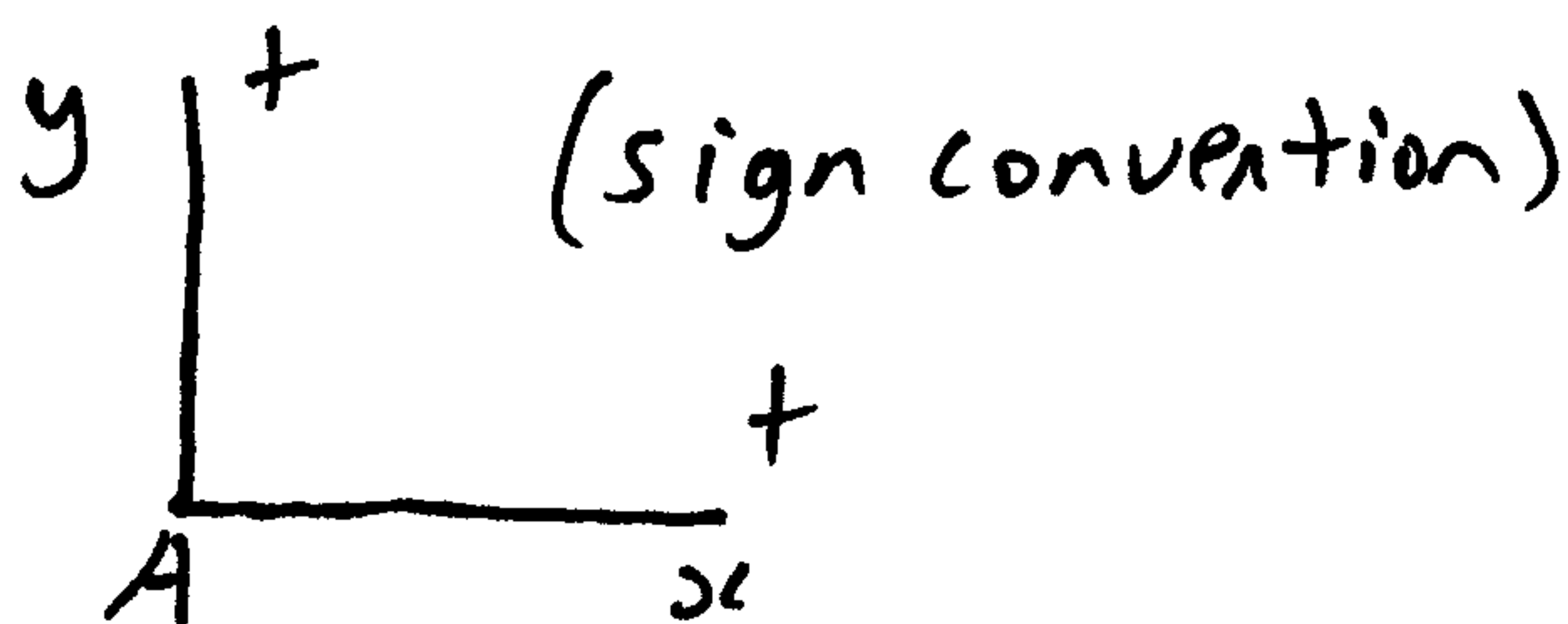


A baseball player throws a ball at an initial speed of  $20 \text{ m/s}$ , from point A, which is  $1.4 \text{ m}$  above the ground and  $14 \text{ m}$  from a wall. What is the launch angle  $\theta$  so that the height  $h$ , which is the distance from the ground to the point of impact on the wall, is a maximum?

Solution: Assumption: - Air resistance is negligible

Set up an  $xy$  coordinate frame with origin at point A, as shown:



The equation for horizontal motion is:

$$d_x = (v_0 \cos \theta) t \quad (1)$$

$$d_x = 14 \text{ m} \quad (\text{horizontal displacement of ball when it hits the wall})$$

$$v_0 = 20 \text{ m/s} \quad (\text{magnitude of initial ball velocity})$$

$$t = ? \quad (\text{time it takes for the ball to hit the wall})$$

The equation for vertical motion is:

$$d_y = (v_0 \sin \theta)t - \frac{1}{2}gt^2 \quad (2)$$

$d_y = ?$  (vertical displacement of ball when it hits the wall - maximize this)

$$g = 9.8 \text{ m/s}^2$$

From equation (1),  $t = \frac{d_x}{v_0 \cos \theta} = \frac{14}{20 \cos \theta}$

Substitute this into equation (2):

$$d_y = d_x \tan \theta - \frac{1}{2}g \left( \frac{d_x}{v_0 \cos \theta} \right)^2$$

$$d_y = 14 \tan \theta - \frac{2.4}{(\cos \theta)^2}$$

Find the value of  $\theta$  so that  $d_y$  is a maximum. The easiest way to do this is to try out different values of  $\theta$ , and using trial and error determine for which  $\theta$  the value of  $d_y$  is maximum.

maximum  $d_y = 18.0 \text{ m}$   
when  $\theta = 71^\circ$

Therefore, maximum  $h = 18.0 + 1.4 = 19.4 \text{ m}$   
when  $\theta = 71^\circ$  (answer)