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This is a problem involving projectile motion.

- A ball is launched from the ground into the air. At a height of 7.3 m, the velocity of the ball is observed to be $\vec{v} = 8.2\hat{i} + 5.7\hat{j}$ in meters per second.

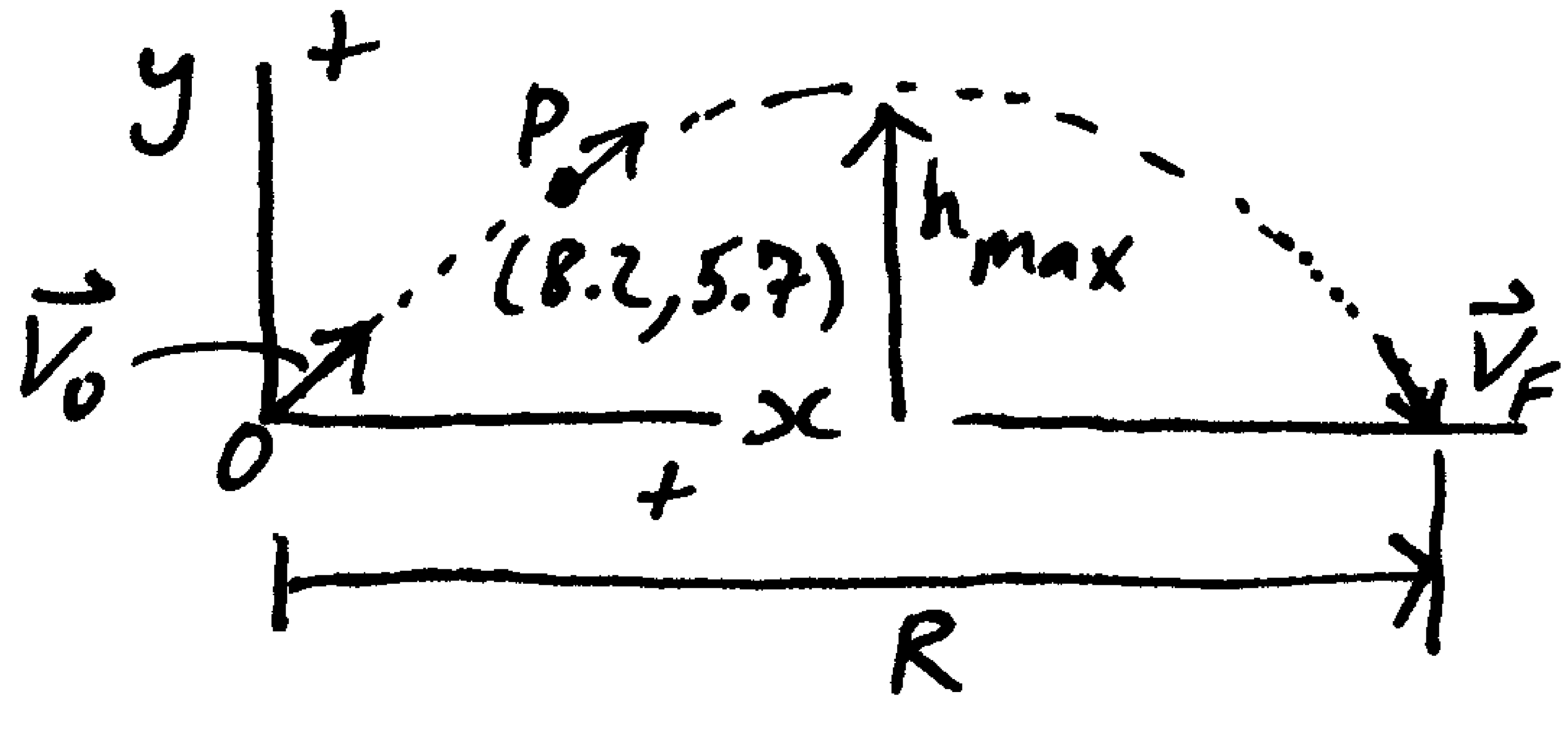
(a) What is the maximum height reached by the ball?

(b) What will be the total horizontal distance traveled by the ball?

(c) At the instant just before the ball hits the ground, what is the magnitude and direction of its velocity?

Solution: Assumption: - Air resistance is negligible

Set up an xy coordinate frame with origin at the launch location of the ball, as shown:



The equation for horizontal motion is:

$$d_x = v_{0x} t \qquad (1)$$

when ball is observed, at point P

$d_x = ?$ (horizontal displacement of ball)

$v_{ox} = ?$ (magnitude of initial ball velocity in horizontal direction)

$t = ?$ (air borne time of ball)

The equation for vertical motion is:

$d_y = v_{oy} t - \frac{1}{2} g t^2$ (2)

$d_y = 7.3\,m$ (vertical displacement of ball, at point P)

$v_{oy} = ?$ (magnitude of initial ball velocity in vertical direction)

$g = 9.8\,m/s^2$

Substitute:

$$7.3 = v_{oy} t - \frac{1}{2}(9.8) t^2$$

(2) $\Rightarrow$ $7.3 = v_{oy} t - 4.9 t^2$

The equation for horizontal velocity is:

$v_x = v_{ox}$ (horizontal velocity stays the same)

$v_x = 8.2\,m/s$ (as observed at point P)

Therefore, equation (1) $\Rightarrow$ $d_x = 8.2t$

The equation for vertical velocity is:

$$v_y = v_{oy} - gt \quad (3)$$

$$v_y = v_{oy} - 9.8t$$

$v_y = 5.7\ m/s$ (as observed at point P)

$$(3) \Rightarrow 5.7 = v_{oy} - 9.8t$$

$$v_{oy} = 5.7 + 9.8t$$

Substitute this into equation (2):

$$7.3 = (5.7 + 9.8t)t - 4.9t^2$$

$$7.3 = 5.7t + 4.9t^2$$

$$4.9t^2 + 5.7t - 7.3 = 0$$

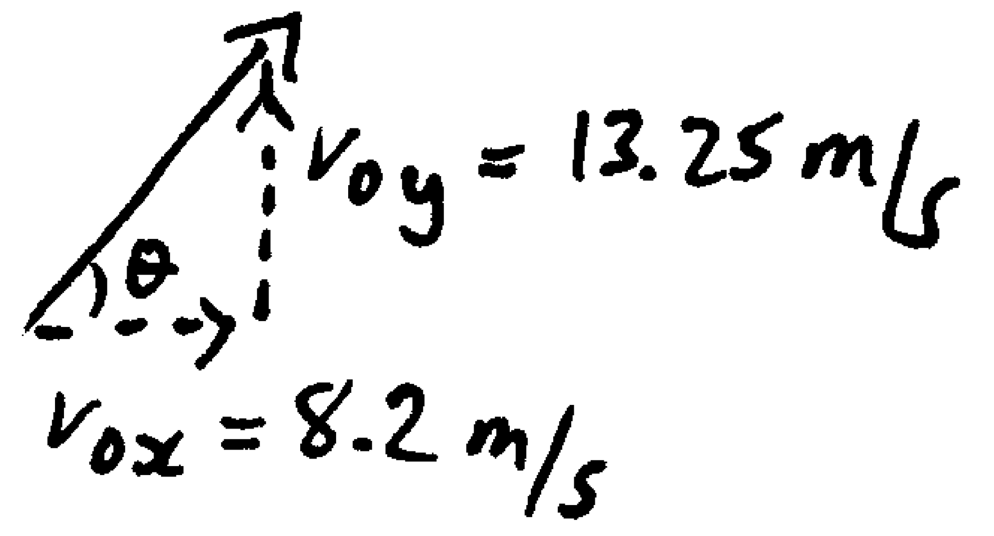
This is a quadratic equation. Solve for t:

$$t = \underbrace{-1.93s}_{\text{not valid}} \quad \text{or} \quad t = 0.77s$$

Therefore, $v_{oy} = 5.7 + 9.8(0.77) = 13.25\ m/s$

and from before, $v_{ox} = 8.2\ m/s$

At launch:

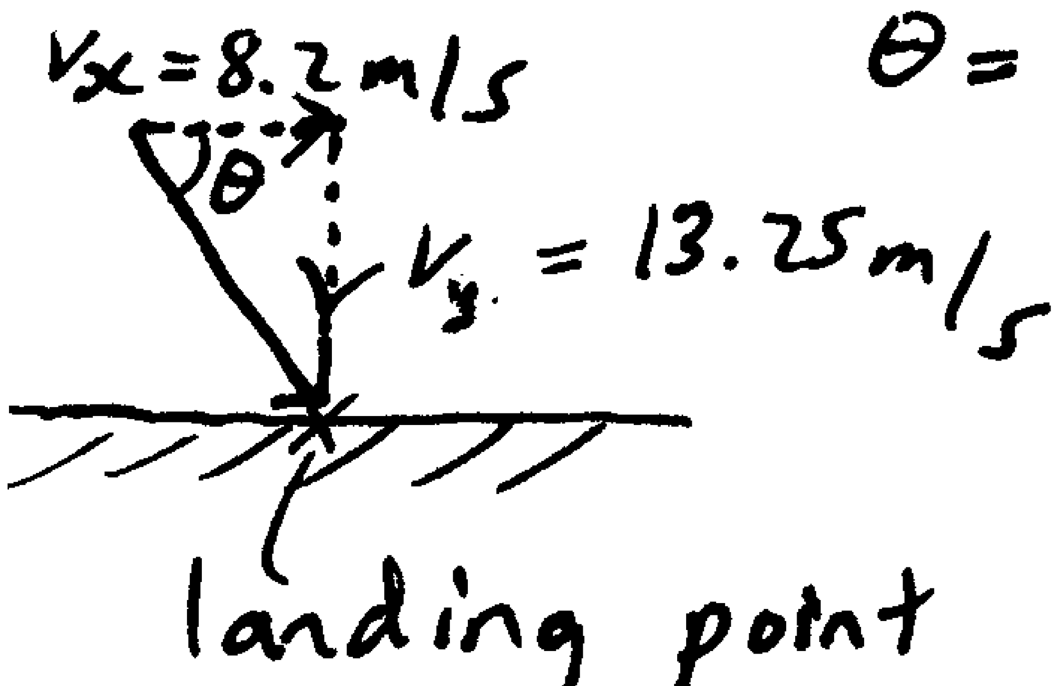


$$\theta = \tan^{-1}\left(\frac{13.25}{8.2}\right)$$

$$\theta = 58.2°$$

Since the ball lands at the same elevation as during launch, the vertical velocity has the same magnitude but opposite direction:

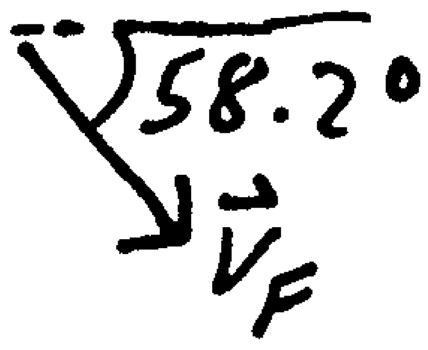
$\theta = 58.2°$



During landing, the magnitude of the velocity is:

$$|\vec{V}_F| = \sqrt{(8.2)^2 + (13.25)^2} = 15.6 \text{ m/s}$$

and the direction is 58.2° below the horizonal



(answer for part (c))

(a) At peak height, $v_y = 0$

From equation (3):

$$v_y = v_{oy} - gt$$

$$v_y = 13.25 - 9.8t$$

$$0 = 13.25 - 9.8t, \quad t = 1.35 \text{ s}$$

From equation (2): $d_y = v_{oy}t - \frac{1}{2}gt^2$

At peak height: $d_y = (13.25)(1.35) - \frac{1}{2}(9.8)(1.35)^2$

$d_y = 8.96$ m (answer)

(b) When the ball hits the ground, $d_y = 0$.

From equation (2):

$$d_y = V_{oy} t - \frac{1}{2} g t^2$$

$$0 = V_{oy} t - \frac{1}{2} g t^2$$

$$0 = V_{oy} - \frac{1}{2} g t$$

$$t = \frac{2 V_{oy}}{g} = \frac{2(13.25)}{9.8} = 2.7\,s$$

From equation (1): $d_x = V_{ox} t$, $t = 2.7\,s$

$$d_x = (8.2\,m/s)(2.7\,s) = 22.2\,m$$

$= R$ (answer)

This is the horizontal travel distance of the ball