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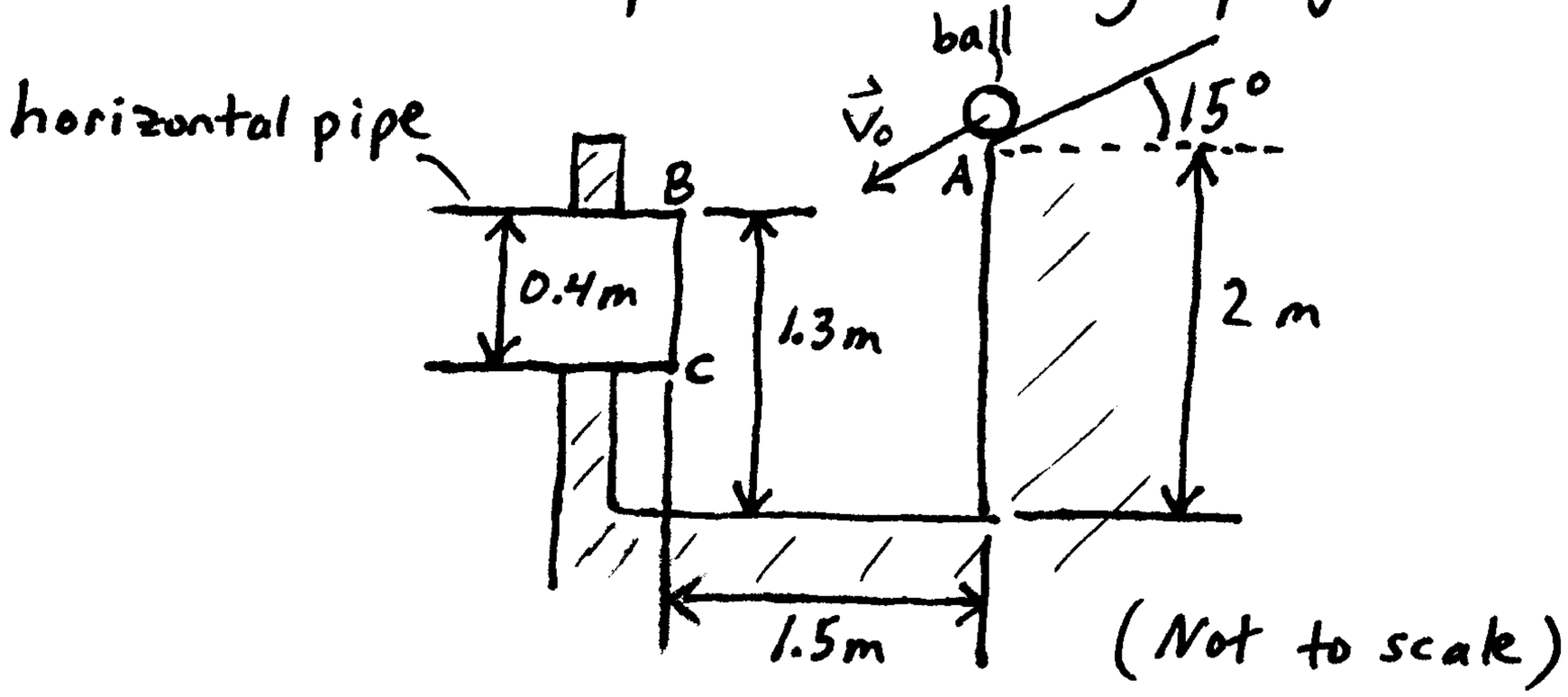
This is a problem involving projectile motion.



A ball rolls down a ramp that is inclined at 15° with the horizontal. At the edge of the ramp (point A), the velocity of the ball is $\vec{v}_0$. Determine the range of values of v_0 so that the ball enters the horizontal pipe shown.

Solution: Assumptions: - Ignore dimensions of the ball
- Air resistance is negligible

Set up an xy coordinate frame with origin at point A, as shown:

The equation for horizontal motion is:

$$d_x = (v_0 \cos 15°)\, t \qquad (1)$$

$d_x = 1.5\,m$ (horizontal displacement when ball enters the pipe)

$v_0 = ?$ (magnitude of initial ball velocity)

$t = ?$ (time it takes the ball to cover the horizontal displacement d_x)

The equation for vertical motion is:

$$d_y = (v_0 \sin 15°)t + \frac{1}{2}gt^2 \qquad (2)$$

$g = 9.8\,m/s^2$

$d_y = 2 - 1.3 = 0.7\,m$ (vertical displacement when ball enters the top of the pipe at point B)

or

$d_y = 2 - (1.3 - 0.4)$
$= 1.1\,m$ (vertical displacement when ball enters the bottom of the pipe at point C)

From equation (1), $t = \dfrac{d_x}{v_0 \cos 15°} = \dfrac{1.5}{v_0 \cos 15°}$

Substitute this into equation (2):

$$d_y = 1.5 \tan 15° + \frac{1}{2}(9.8)\left(\frac{1.5}{v_0 \cos 15°}\right)^2$$

Solve:

$$v_0 = \left(\frac{1.5}{\cos 15°}\right)\sqrt{\frac{4.9}{d_y - 1.5 \tan 15°}} \qquad (ANS.)$$

For $d_y = 0.7\,m$, $v_0 = 6.3\,m/s$ (upper limit). For $d_y = 1.1\,m$, $v_0 = 4.1\,m/s$ (lower limit)