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This is a 3-D problem involving instantaneous acceleration.

A particle is initially located at the origin and has an initial velocity of $\vec{v} = 3.0\hat{i} + 2.0\hat{j} - 1.0\hat{k}$, in meters per second. It experiences a constant acceleration of $\vec{a} = -1.0\hat{i} - 1.5\hat{j}$, in meters per second squared.

(a) What is the velocity of the particle when its y-coordinate is a maximum?

(b) Where is the particle located at this instant?

Solution:

(a) y-coordinate: $y = \underbrace{2.0}_{v_y \text{ (initial)}} t + \frac{1}{2}\underbrace{(-1.5)}_{\text{acceleration, } a_y} t^2$

Maximum y occurs when $v_y = 0$

$$\Rightarrow v_y = 2.0 + (-1.5)t = 0$$
$$t = 1.33 \text{ s}$$

Maximum $y = 2.0(1.33) - \frac{1}{2}(1.5)(1.33)^2$
$= 1.33 \text{ m}$

$$v_x = \underbrace{3.0}_{v_x \text{ (initial)}} + \underbrace{(-1.0)}_{a_x} t$$

At $t = 1.33 \text{ s}$, $v_x = 1.67 \text{ m/s}$

v_z does not change since $a_z = 0$.

Therefore, when the y-coordinate is a maximum:
$$\vec{v} = 1.67\hat{i} - 1.0\hat{k} \quad \text{(answer)}$$

(b) From part (a), $y = 1.33\,m$

$$z = \underbrace{-1.0}_{v_z}t = -1.0(1.33) = -1.33\,m$$

$$x = 3.0t + \frac{1}{2}(-1.0)t^2$$

$$x = 3.0(1.33) + \frac{1}{2}(-1.0)(1.33)^2$$

$$x = 3.1\,m$$

Therefore, the particle is located at:

$$\underbrace{(3.1, 1.33, -1.33)}_{\text{units in meters.}} \quad \text{(answer)}$$