real-world-physics-problems.com
Jan. 25, 2020

This is a 1-D problem involving instantaneous velocity and speed.

A particle's position is given by $x = 6 + 4t - 12t^2 + 6t^3$, where t is in seconds and x is in meters.

(a) At what time is the velocity of the particle equal to $-1.5\ m/s$?

(b) At what time is the speed of the particle equal to $1.5\ m/s$?

(c) What is the minimum velocity and minimum speed of the particle?

Solution:

(a) $v = \frac{dx}{dt} = 4 - 24t + 18t^2$

Solve for t when $v = -1.5\ m/s$

$-1.5 = 4 - 24t + 18t^2$

$18t^2 - 24t + 5.5 = 0$

This is a quadratic equation.

$$t = \frac{24 \pm \sqrt{24^2 - 4(18)(5.5)}}{36}$$

$t = 0.294\ s$ or $t = 1.039\ s$ (answer)

(b) We must solve for t when $s = |v| = 1.5\,m/s$.

So, $v = -1.5\,m/s$ or $v = 1.5\,m/s$

For $v = -1.5\,m/s$, $t = 0.294\,s$ or $t = 1.039\,s$ (answer)
(from (a))

For $v = 1.5\,m/s$, solve for t:

$v = 4 - 24t + 18t^2$ (from (a))

$1.5 = 4 - 24t + 18t^2$

$18t^2 - 24t + 2.5 = 0$

This is a quadratic equation.

$$t = \frac{24 \pm \sqrt{24^2 - 4(18)(2.5)}}{36}$$

$t = 1.219\,s$ or $t = 0.114\,s$ (answer)

There are 4 times in total where the speed is $1.5\,m/s$. They are: $0.114\,s$, $0.294\,s$, $1.039\,s$, $1.219\,s$

(c) From (a), $v = 4 - 24t + 18t^2$

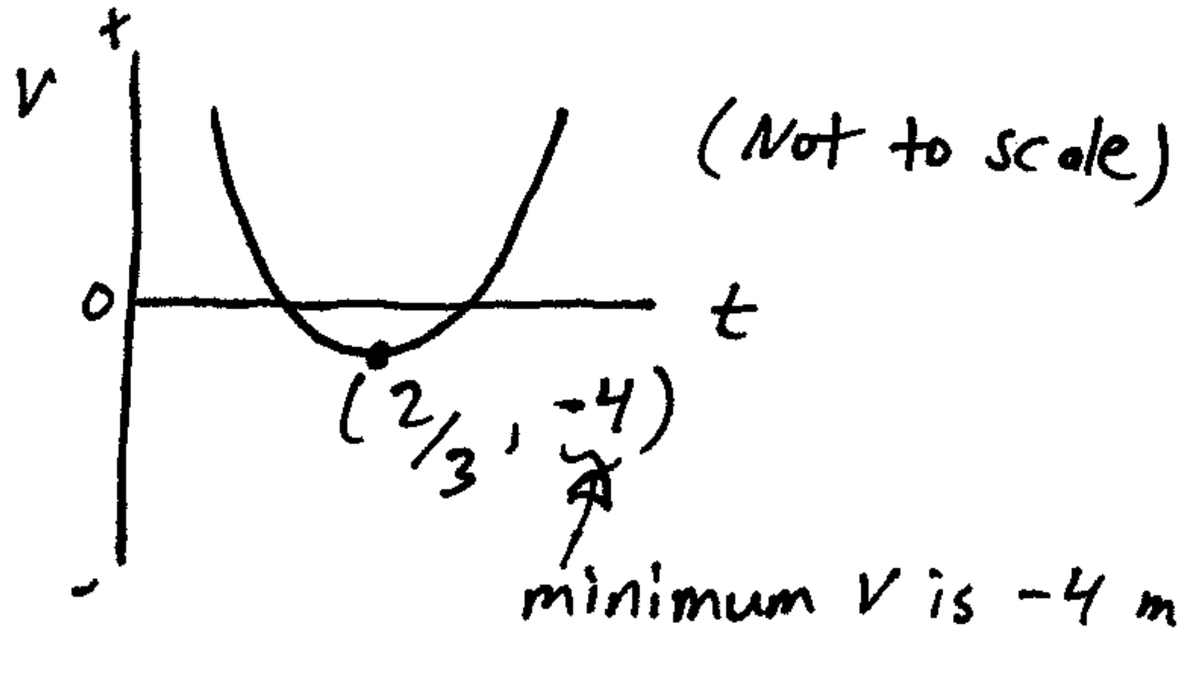
Complete the square to find the vertex and this will give the minimum velocity of the particle:

$$v = 18\left(t^2 - \frac{24}{18}t + \frac{4}{9} - \frac{4}{9}\right) + 4$$

$$v = 18\left(t - \tfrac{2}{3}\right)^2 - 4$$

Therefore, the minimum velocity is $-4\,m/s$, and the minimum speed is 0, since $s = |v|$.

A sketch of v vs. t would look like this:



A sketch of s vs. t would look like this: