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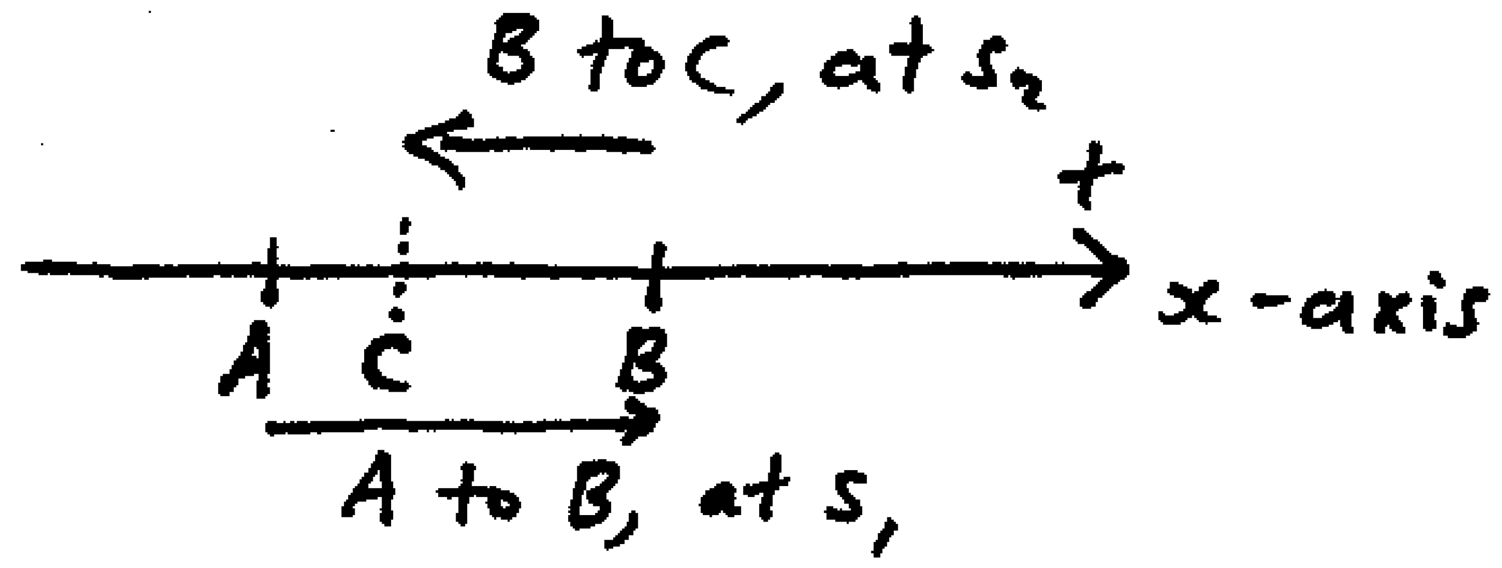
This is a 1-D problem involving average velocity and average speed.

An object moves along the positive x-axis, to the right, at speed s_1, from point A to point B. It then moves at speed s_2, from point B to point C. If point C is located to the left of point B, derive an equation relating the average velocity of the object to the average speed of the object, between points A and C.

Solution:



Let d_1 be the distance between A and B.
Let d_2 be the distance between B and C.

Time interval between A and C: $\Delta t = \frac{d_1}{s_1} + \frac{d_2}{s_2}$

Displacement between A and C $= d_1 - d_2 = \Delta x$

Total travel distance between A and C $= d_1 + d_2$

Average velocity between A and C: $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{d_1 - d_2}{\frac{d_1}{s_1} + \frac{d_2}{s_2}}$

Average speed between A and C: $\bar{s} = \frac{\text{Total distance}}{\Delta t} = \frac{d_1 + d_2}{\frac{d_1}{s_1} + \frac{d_2}{s_2}}$

Rewrite $\bar{v}$ and $\bar{s}$:

$$\bar{v} = \frac{\frac{1}{d_1}(d_1 - d_2)}{\frac{1}{d_1}\left(\frac{d_1}{s_1} + \frac{d_2}{s_2}\right)} = \frac{1 - \frac{d_2}{d_1}}{\frac{1}{s_1} + \left(\frac{d_2}{d_1}\right)\cdot\frac{1}{s_2}} \quad (1)$$

$$\bar{s} = \frac{\frac{1}{d_1}(d_1 + d_2)}{\frac{1}{d_1}\left(\frac{d_1}{s_1} + \frac{d_2}{s_2}\right)} = \frac{1 + \frac{d_2}{d_1}}{\frac{1}{s_1} + \left(\frac{d_2}{d_1}\right)\cdot\frac{1}{s_2}} \quad (2)$$

From (1), after some algebraic manipulation,

$$\frac{d_2}{d_1} = \frac{1 - \frac{\bar{v}}{s_1}}{\frac{\bar{v}}{s_2} + 1} \quad (3)$$

Similarly, from (2), $\frac{d_2}{d_1} = \frac{1 - \frac{\bar{s}}{s_1}}{\frac{\bar{s}}{s_2} - 1}$ (4)

(3) is equal to (4), therefore,

$$\frac{1 - \frac{\bar{v}}{s_1}}{\frac{\bar{v}}{s_2} + 1} = \frac{1 - \frac{\bar{s}}{s_1}}{\frac{\bar{s}}{s_2} - 1}$$

Note: If $s_1 = s_2$ this equation won't work for solving $\bar{v}$, so must use (1) knowing d_1 and d_2.

After some algebraic manipulation, this simplifies to: $\bar{v}\left(\frac{1}{s_1} - \frac{1}{s_2}\right) + \bar{s}\left(\frac{1}{s_1} + \frac{1}{s_2}\right) = 2$ (ans.)