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This is a 1-D problem involving average velocity and average speed.

A particle is moving along the x-axis at a speed defined by $s(t) = at + b$, where a and b are constants and t is time. If the particle starts moving at time t_1 and stops moving at time t_2, what is the value of a and b so that the average velocity and average speed, between t_1 and t_2, is equal to

$$\frac{s(t_1) + s(t_2)}{2}.$$

Solution:

$$s(t_1) = at_1 + b \quad \text{and} \quad \frac{s(t_1) + s(t_2)}{2}$$
$$s(t_2) = at_2 + b \qquad\qquad = \frac{a(t_1 + t_2) + 2b}{2} \quad (1)$$

The distance travelled along the x-axis is also equal to the displacement of the particle (but is not true in general).

$$\text{Distance travelled} = \int_{t_1}^{t_2} s(t)\,dt$$

$$= \int_{t_1}^{t_2} (at + b)\,dt = \frac{at^2}{2} + bt \Big|_{t_1}^{t_2}$$

$$= \frac{a}{2}(t_2^2 - t_1^2) + b(t_2 - t_1)$$

The average velocity is: $\frac{\text{Displacement}}{\Delta t} = \bar{v}$

Displacement $= \frac{a}{2}(t_2^2 - t_1^2) + b(t_2 - t_1)$

Time interval, $\Delta t = t_2 - t_1$

Therefore, $\bar{v} = \frac{\frac{a}{2}(t_2^2 - t_1^2) + b(t_2 - t_1)}{t_2 - t_1}$

$$\bar{v} = \frac{a}{2}(t_2 + t_1) + b$$

→ This is equal to equation (1) for all values of a and b. (answer)

The average speed is: $\frac{\text{Distance}}{\Delta t} = \bar{s}$

Since distance = displacement, then once more, average speed ($\bar{s}$) is equal to equation (1) for all values of a and b. (answer)

Note that displacement is a vector quantity and velocity is also a vector quantity. But since this is a 1-D problem, we don't have to consider vector quantities in the calculations.