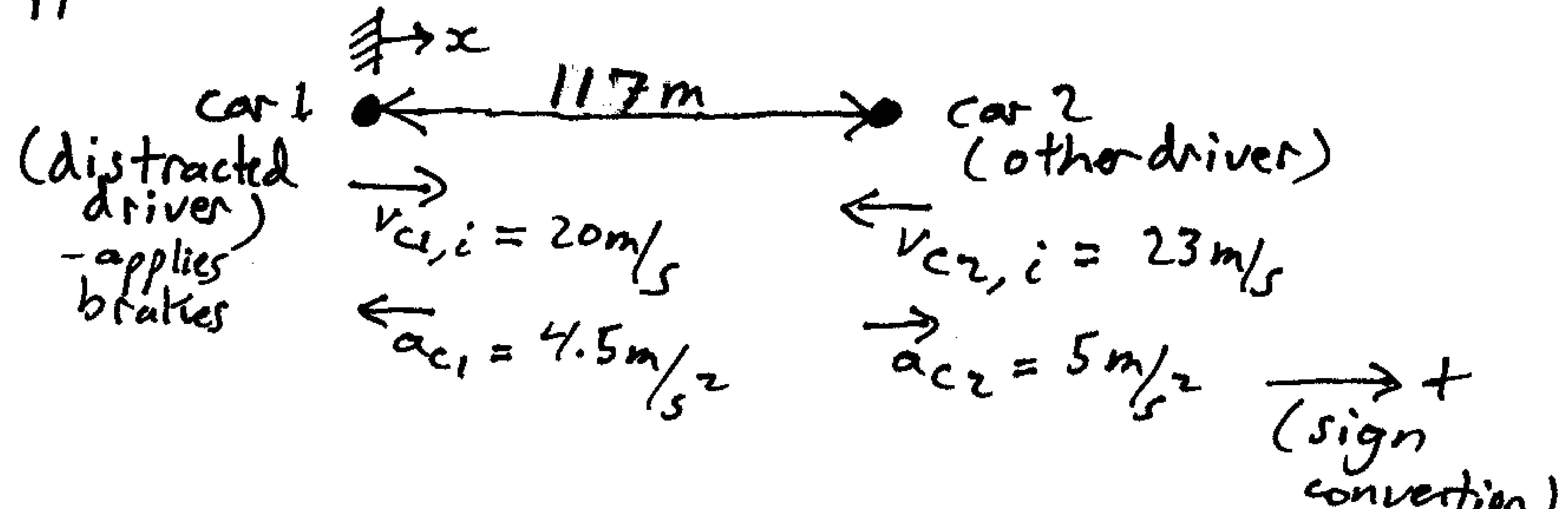
This is a 1-D problem involving constant acceleration.

A distracted driver is driving on the wrong side of the road, when he notices an oncoming vehicle moving towards him. He quickly applies the brakes, causing his car to decelerate at $4.5\,m/s^2$, from an initial speed of $20\,m/s$. The other driver also applies the brakes, 1.1 seconds later, causing his car to decelerate at $5\,m/s^2$, from an initial speed of $23\,m/s$. At the instant the distracted driver applies the brakes, the front of each car is separated by a distance of 117 m. Is there a collision between the two cars?

Solution:

Set up a coordinate frame with origin at the location of the distracted driver's car (but this choice is completely up to you), at the instant he applies the brakes.



The position of car 1 is:

$$x_{c1} = v_{c1,i}\,t - \frac{1}{2} a_{c1} t^2,$$

where t is the time from when the brakes are applied

For $0 \le t \le 1.1$, the position of car 2 is:

$$x_{c2} = 117 - v_{c2,i}\, t$$

For $t > 1.1$, the position of car 2 is:

$$x_{c2} = \underbrace{117 - v_{c2,i}(1.1)}_{\text{position of car 2 at instant it starts decelerating}} - \underbrace{\left(v_{c2,i}(t-1.1) - \tfrac{1}{2}a_{c2}(t-1.1)^2\right)}_{\text{travel distance of car 2 after it starts decelerating.}}$$

Note that we must use $(t-1.1)$, and not t, since car 2 deceleration starts at a time that is offset by 1.1 seconds from when car 1 starts braking

Simplify:

$$x_{c2} = 117 - v_{c2,i}\, t + \tfrac{1}{2}a_{c2}(t-1.1)^2$$

Check the separation distance between both cars at $t = 1.1$ seconds.

$$x_{c2} - x_{c1} = 117 - 23(1.1) - \left(20(1.1) - \tfrac{1}{2}(4.5)(1.1)^2\right)$$

$x_{c2} - x_{c1} = 72.4\ m$ (no collision - good - now let's check if there's a collision for $t > 1.1\,s$)

First, let's check the position of each car assuming each comes to a complete stop, with no collision, which we must then check.

The time it takes car 1 to come to a complete stop is: $t_{c1} = \dfrac{v_{c1,i}}{a_{c1}} = \dfrac{20}{4.5} = 4.44\ s$

The position of car 1 at the instant it stops is:

$$x_{c1} = 20(4.44) - \frac{1}{2}(4.5)(4.44)^2$$

$$x_{c1} = 44.4 \text{ m}$$

The time it takes car 2 to come to a complete stop is: $t_{c2} = \frac{v_{c2,i}}{a_{c2}} = \frac{23}{5} = 4.6 \text{ s}$

The position of car 2 at the instant it stops is:

$$x_{c2} = 117 - 23(4.6) + \frac{1}{2}(5)(4.6 - 1.1)^2$$

$$x_{c2} = 41.8 \text{ m}$$

Since $x_{c1} > x_{c2}$ there is a collision, but fortunately at low speed, so that only some vehicle damage will result, leaving the drivers uninjured.

Since $x_{c1} > x_{c2}$ when each car comes to a complete stop (in theory, not in reality), means that at least one car is still moving when $x_{c1} = x_{c2}$ (the point of collision). The small difference between x_{c1} and x_{c2} (44.4 m and 41.8 m, respectively) means that the collision speed is low.