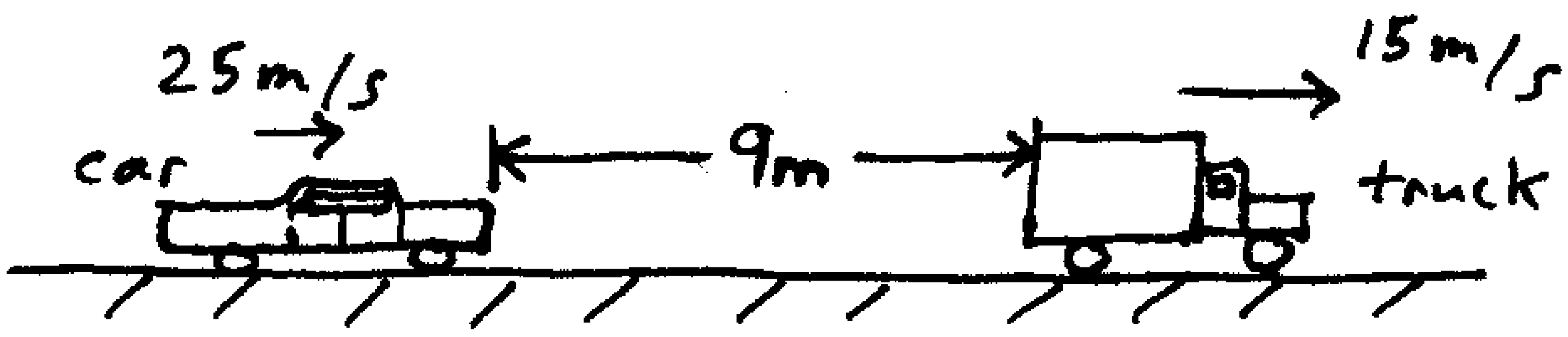
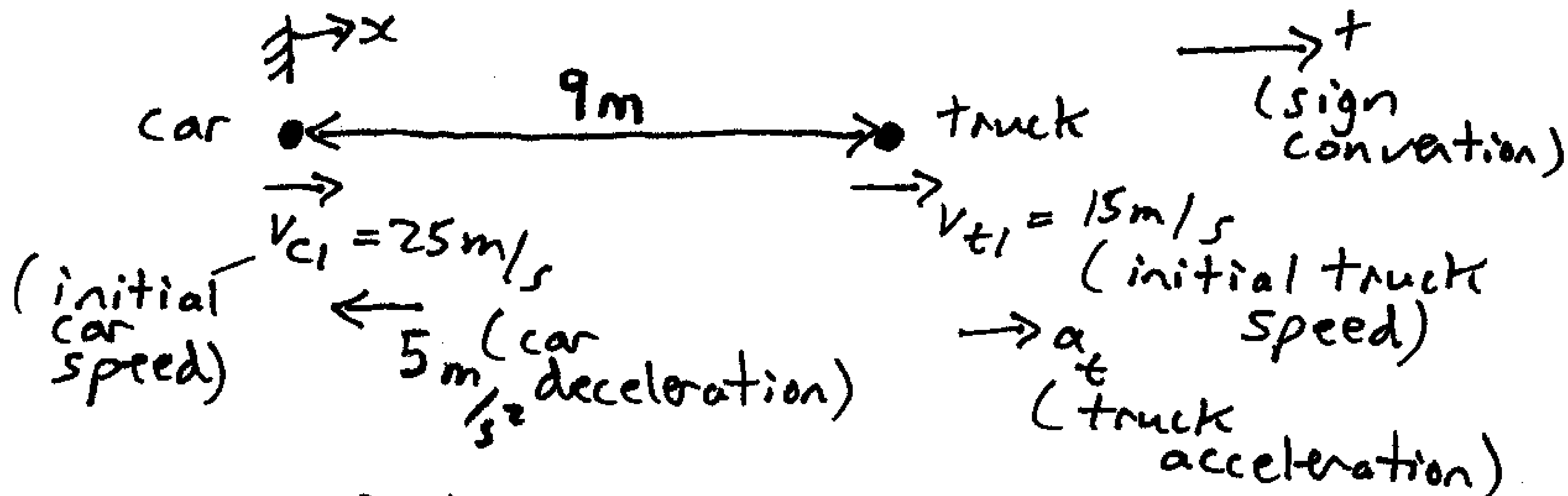
This is a 1-D problem involving constant acceleration.



A distracted driver is cruising at 25 m/s when he suddenly notices a truck directly ahead moving at 15 m/s, in the same direction. At the instant he brakes, the distance between the front of the car and the back of the truck is 9m. The car decelerates at 5 m/s^2, and one second after the driver of the car applies the brakes, the truck driver notices the car behind and starts accelerating at a_t. What is the minimum value of a_t in order to avoid a collision?

<u>Solution:</u>

Set up a coordinate frame with origin at the car location at the instant the brakes are applied.



The position of the car is:

$x_c = v_{c1}t - \frac{1}{2}(5)t^2$, where t is the time from when the brakes are applied.

For $0 \le t \le 1$, the position of the truck is:

$$x_t = 9 + v_{t1} t$$ (truck driver doesn't notice car behind him and continues at same speed)

For $t > 1$, the position of the truck is:

$$x_t = \underbrace{9 + v_{t1}(1)}_{\text{position of truck at instant truck driver starts accelerating}} + \underbrace{v_{t1}(t-1) + \frac{1}{2} a_t (t-1)^2}$$

Note that we must use $(t-1)$, and not t, since truck acceleration starts at a time that is offset by 1 second from when the car starts braking

Simplify:

$$x_t = 9 + v_{t1} t + \frac{1}{2} a_t (t-1)^2$$

Check the separation distance between car and truck at $t = 1$ second:

$$x_t - x_c = 9 + 15(1) - \left(25(1) - \frac{1}{2}(5)(1)^2\right)$$

$$x_t - x_c = 1.5\,m$$ (no collision - good - this means that the truck has the possibility to accelerate out of the way)

Now, for $t > 1$ second,

$$x_t - x_c = 9 + 15t + \frac{1}{2} a_t (t-1)^2 - \left(25t - \frac{1}{2}(5)t^2\right)$$

Simplify:

$$x_t - x_c = 9 - 10t + \left(\frac{1}{2} a_t + 2.5\right)t^2 - a_t t + \frac{1}{2} a_t$$

$$x_t - x_c = 9 + \frac{1}{2} a_t - (10 + a_t)t + (\tfrac{1}{2} a_t + 2.5)t^2$$

This is a quadratic equation and we can complete the square:

$$x_t - x_c = \left(\tfrac{1}{2}a_t + 2.5\right)t^2 - (10 + a_t)t + 9 + \tfrac{1}{2}a_t$$

$$x_t - x_c = \left(\tfrac{1}{2}a_t + 2.5\right)\left(t^2 - \frac{(10 + a_t)\,t}{\left(\tfrac{1}{2}a_t + 2.5\right)}\right) + 9 + \tfrac{1}{2}a_t$$

$$x_t - x_c = \left(\tfrac{1}{2}a_t + 2.5\right)\left(t^2 - \frac{(10 + a_t)t}{\left(\tfrac{1}{2}a_t + 2.5\right)} + \frac{(10 + a_t)^2}{4\left(\tfrac{1}{2}a_t + 2.5\right)^2} - \frac{(10 + a_t)^2}{4\left(\tfrac{1}{2}a_t + 2.5\right)^2}\right) + 9 + \tfrac{1}{2}a_t$$

$$x_t - x_c = \left(\tfrac{1}{2}a_t + 2.5\right)\left(t - \frac{(10 + a_t)}{2\left(\tfrac{1}{2}a_t + 2.5\right)}\right)^2 - \frac{(10 + a_t)^2}{4\left(\tfrac{1}{2}a_t + 2.5\right)} + 9 + \tfrac{1}{2}a_t$$

The minimum value of a_t in order to avoid a collision corresponds to a minimum separation distance equal to zero (so that the car and truck are barely touching)

The above quadratic equation has a minimum value of zero when:

$$\frac{-(10 + a_t)^2}{4\left(\tfrac{1}{2}a_t + 2.5\right)} + 9 + \tfrac{1}{2}a_t = 0$$

Simplify, and the above equation becomes:

$$\frac{-(10+a_t)^2 + (9+\frac{1}{2}a_t)(2a_t+10)}{4(\frac{1}{2}a_t+2.5)} = 0$$

$$\Rightarrow \frac{-(a_t^2+20a_t+100)+(18a_t+90+a_t^2+5a_t)}{4(\frac{1}{2}a_t+2.5)} = 0$$

$$\Rightarrow \frac{3a_t-10}{4(\frac{1}{2}a_t+2.5)} = 0$$

$$\Rightarrow 3a_t - 10 = 0$$

$$a_t = \frac{10}{3}\ m/s^2$$ (answer)

This is the minimum truck acceleration to avoid a collision.