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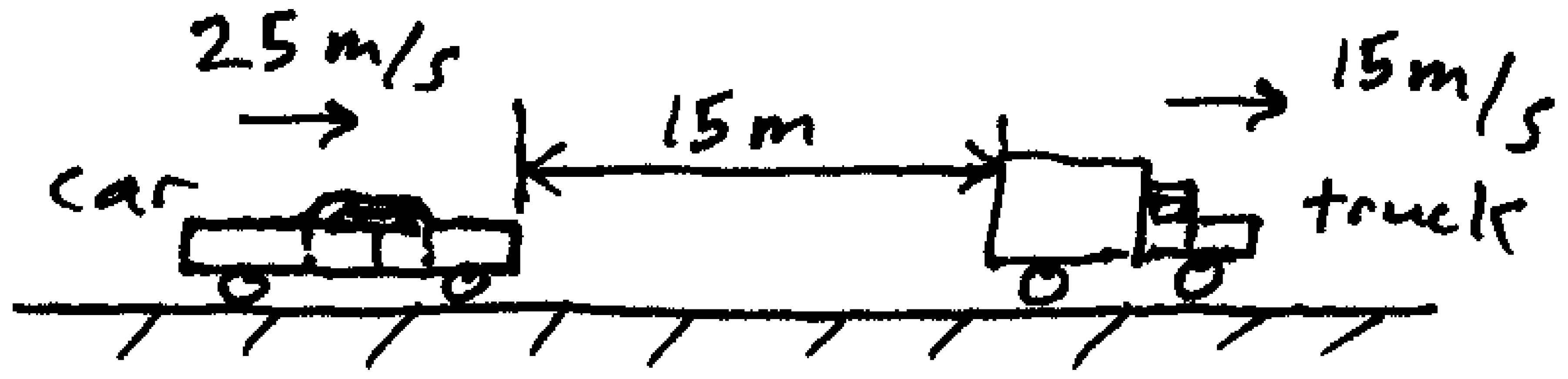
Jan. 19, 2020

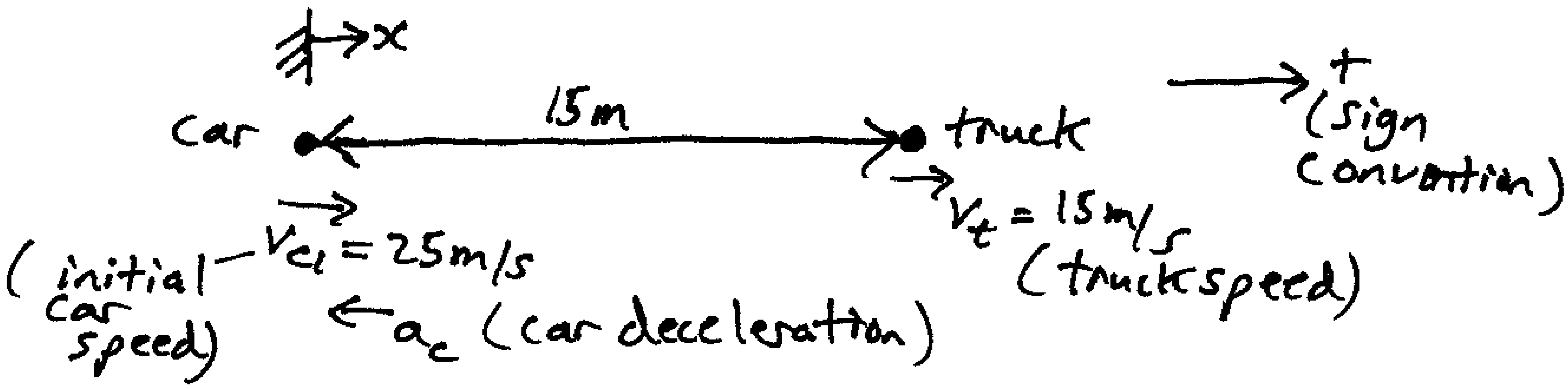
This is a 1-D problem involving constant acceleration.



A distracted driver is cruising at 25 m/s when he suddenly notices a truck directly ahead moving at 15 m/s, in the same direction. At the instant he brakes, the distance between the front of the car and the back of the truck is 15 m. If the car decelerates at a_c, and the truck maintains its speed, what is the minimum value of a_c in order to avoid a collision?

Solution:

Set up a coordinate frame with origin at the car location at the instant the brakes are applied.



The position of the car is:

$$x_c = v_{c1}t - \frac{1}{2}a_c t^2,$$ where t is the time from when the brakes are applied

The position of the truck is:

$$x_t = 15 + v_t t$$

The separation distance between the car and truck is:

$$x_t - x_c = 15 + v_t t - (v_{c1} t - \frac{1}{2} a_c t^2)$$

The minimum value of a_c in order to avoid a collision corresponds to a minimum separation distance equal to zero (so that the car and truck are barely touching).

Now, $x_t - x_c = 15 + (v_t - v_{c1})t + \frac{1}{2} a_c t^2$

Substitute known values:

$$x_t - x_c = 15 + (15 - 25)t + \frac{1}{2} a_c t^2$$

$$x_t - x_c = 15 - 10t + \frac{1}{2} a_c t^2$$

$$x_t - x_c = 15 + \frac{1}{2} a_c t^2 - 10t$$

This is a quadratic equation and we can complete the square:

$$x_t - x_c = 15 + \frac{1}{2} a_c \left(t^2 - \frac{20}{a_c} t + \frac{100}{a_c^2} - \frac{100}{a_c^2}\right)$$

$$x_t - x_c = 15 + \frac{1}{2} a_c \left(t^2 - \frac{20}{a_c} t + \frac{100}{a_c^2}\right) - \frac{50}{a_c}$$

$$x_t - x_c = 15 + \frac{1}{2} a_c \left(t - \frac{10}{a_c}\right)^2 - \frac{50}{a_c}$$

This has a minimum value of zero when $15 - \frac{50}{a_c} = 0$, for which $a_c = 10/3 \ m/s^2$ (answer).

This is the minimum value to avoid a collision.