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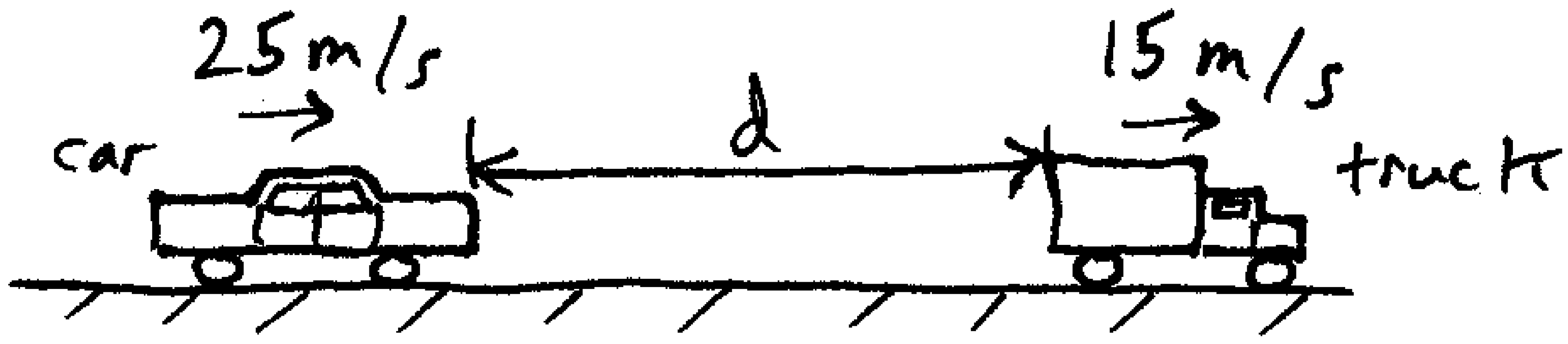
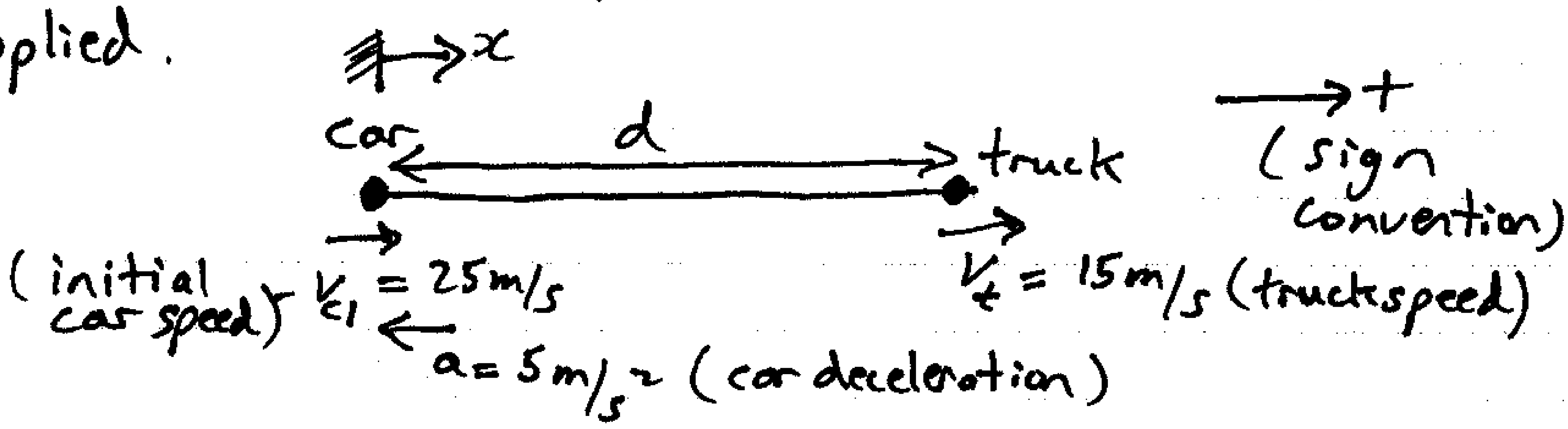
This is a 1-D problem involving constant acceleration.



A distracted driver is cruising at 25 m/s when she suddenly notices a truck directly ahead moving at 15 m/s, in the same direction. At the instant she brakes, the distance between the front of the car and the back of the truck is d. If the car decelerates at 5.0 m/s^2, and the truck maintains its speed, what is the minimum value of d in order to avoid a collision?

Solution:

Set up a coordinate frame with origin at the car location at the instant the brakes are applied.



The position of the car is:

$x_c = v_{c1} t - \frac{1}{2} a t^2$, where t is the time from when the brakes are applied

The position of the truck is:

$$x_t = d + v_t t$$

The separation distance between the car and truck is:

$$x_t - x_c = d + v_t t - (v_{c_1} t - \frac{1}{2} a t^2)$$

The minimum value of d in order to avoid a collision corresponds to a minimum separation distance equal to zero (so that the car and truck are barely touching).

Now, $x_t - x_c = d + (v_t - v_{c_1})t + \frac{1}{2} a t^2$

Substitute known values:

$$x_t - x_c = d + (15 - 25)t + \frac{1}{2}(5)t^2$$

$$x_t - x_c = d - 10t + 2.5t^2$$

$$x_t - x_c = d + 2.5t^2 - 10t$$

This is a quadratic equation and we can complete the square:

$$x_t - x_c = d + 2.5(t^2 - 4t + 4 - 4)$$

$$x_t - x_c = d + 2.5(t^2 - 4t + 4) - 10$$

$$x_t - x_c = d + 2.5(t - 2)^2 - 10$$

This has a minimum value of zero when d has a value of 10 meters

This is the minimum value to avoid a collision. (answer).