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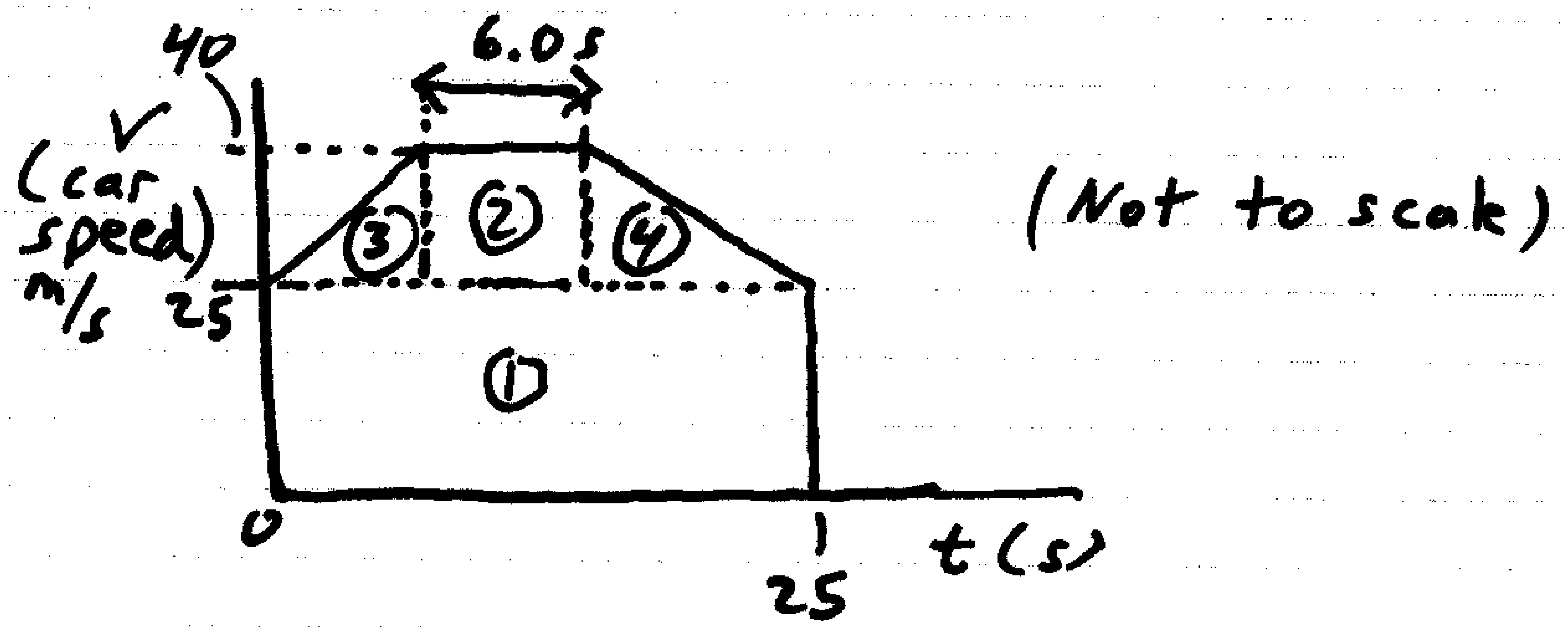
This is a 1-D problem involving constant acceleration.

A car travelling on a straight road at 25 m/s undergoes constant acceleration until it reaches a speed of 40 m/s. The car then maintains this speed for 6.0 seconds. The brakes are then applied causing the car to undergo constant deceleration until it once more reaches a speed of 25 m/s. If it takes the car 25 seconds from the time that it starts accelerating to the time that it slows down to 25 m/s, how far does it travel in this time?

Solution:

The easiest way to solve this problem is graphically:



The area under the v-t graph is the travel distance of the car.

Using geometry, the area under the v-t graph is the area of rectangle ① plus the area of rectangle ② plus the area of triangle ③ plus the area of triangle ④.

The area of rectangle ① is (base)(height)
$= 25 \times 25$

The area of rectangle ②
is (base)(height)
$= 6 \times (40 - 25)$

The area of triangle ③ plus the area of triangle ④ can be thought of as a single triangle with area of $\frac{1}{2}$(base)(height) $= \frac{1}{2}(25-6)(40-25)$

Therefore, the area under the v-t graph is:

$$25 \times 25 + 6 \times (40-25) + \frac{1}{2}(25-6)(40-25)$$
$$= 857.5 \text{ m}$$

Therefore, the car travels 857.5 m (answer).

Alternatively, you can solve this the harder way using kinematic equations, which will of course give you the same answer.

Let a_1 be the acceleration when the car gains speed.

Let $-a_2$ be the acceleration when the car slows down.

Let d_1 be the distance travelled during the acceleration phase, and t_1 the corresponding travel time.

Let d_2 be the distance travelled when the car is moving at 40 m/s, and t_2 the corresponding travel time, $t_2 = 6.0$ s

Let d_3 be the distance travelled when the car is decelerating (slowing down), and t_3 the corresponding travel time.

Then, $d_1 + d_2 + d_3$ is the total travel distance.

Use the following kinematic equations:

distance equation: $d_1 = 25t_1 + \frac{1}{2}a_1 t_1^2$, $40 = 25 + a_1 t_1$ ← speed equation

$\Rightarrow a_1 t_1 = 15 \quad (1)$

and, similarly,

$d_2 = 40t_2 = 40(6)$

$d_3 = 40t_3 - \frac{1}{2}a_2 t_3^2$, $25 = 40 - a_2 t_3 \Rightarrow a_2 t_3 = 15 \quad (2)$

Lastly, $t_1 + t_2 + t_3 = 25$ (3)

Now,

$$d_1 + d_2 + d_3 = 25t_1 + \underbrace{\tfrac{1}{2}a_1 t_1^2}_{\text{sub. eq.(1)}} + 40(6) + 40t_3 - \tfrac{1}{2}\underbrace{a_2 t_3^2}_{\text{sub. eq.(2)}}$$

$$d_1 + d_2 + d_3 = 25t_1 + \tfrac{1}{2}(15)t_1 + 40(6) + 40t_3 - \tfrac{1}{2}(15)\underbrace{t_3}_{\text{sub. eq. (3) for } t_3 \text{ with } t_2 = 6}$$

$$d_1 + d_2 + d_3 = 25t_1 + \tfrac{1}{2}(15)t_1 + 240 + \left(40 - \tfrac{15}{2}\right)(25 - t_1 - 6)$$

Simplify:

$$d_1 + d_2 + d_3 = 32.5t_1 + 240 + 32.5(19 - t_1)$$

$$= 240 + 32.5 \times 19 \quad (t_1 \text{ term cancels out})$$

$$= 857.5 \text{ m} \quad (\text{same answer!})$$