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This is a 1-D problem involving free-fall acceleration.

An object is dropped from rest, and one second before it lands it is at half its initial drop height.

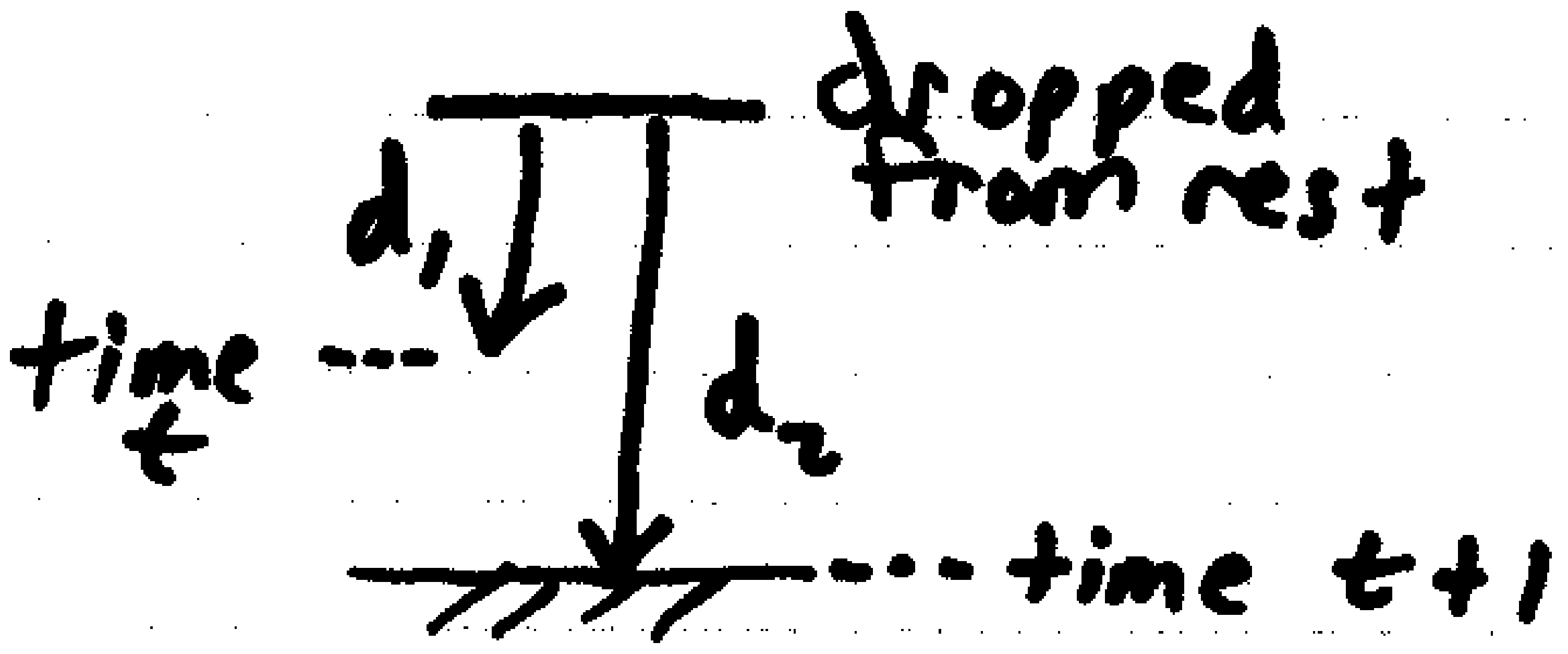
(a) What is the falling time?

(b) What is the drop height?

Solution:

Assumptions: - Object motion is purely vertical
- Air resistance is negligible

(a) Draw a diagram to clarify the problem.



$$\frac{d_2 - d_1}{d_2} = \frac{1}{2} \Rightarrow \frac{d_1}{d_2} = \frac{1}{2}$$

Use the kinematic equation: $d = v_1 t + \frac{1}{2} g t^2$

(1-D equation)

$v_1 = 0$ (starts falling from rest)

$t_1 = ?$ (falling time, minus one second)

$t_1 + 1 =$ falling time

$g = 9.8 \, m/s^2$

$\downarrow +$ (sign convention)

displacement { d_1 = falling distance one second before landing

d_2 = total falling distance, which is the drop height

Now,

$$d_1 = \overset{\nearrow 0}{v_1 t_1} + \frac{1}{2} g t_1^2$$

and

$$d_2 = \overset{\nearrow 0}{v_1}(t_1 + 1) + \frac{1}{2} g (t_1 + 1)^2$$

and

$$\frac{d_1}{d_2} = \frac{1}{2}$$

Substitute:

$$\frac{\frac{1}{2} g t_1^2}{\frac{1}{2} g (t_1 + 1)^2} = \frac{1}{2}$$

Simplify: $$\frac{t_1^2}{(t_1+1)^2} = \frac{1}{2}$$

After some algebra this becomes the quadratic equation: $t_1^2 - 2t_1 - 1 = 0$

Solve:

$t_1 = 1 + \sqrt{2}$ or $t_1 = 1 - \sqrt{2} < 0$ (invalid)

The falling time is $t_1 + 1 = 2 + \sqrt{2} = 3.4$ s (answer)

(b) The drop height is

$$d_2 = \frac{1}{2} g (t_1 + 1)^2 = \frac{1}{2}(9.8)(2+\sqrt{2})^2$$

$= 57.1$ m (answer)