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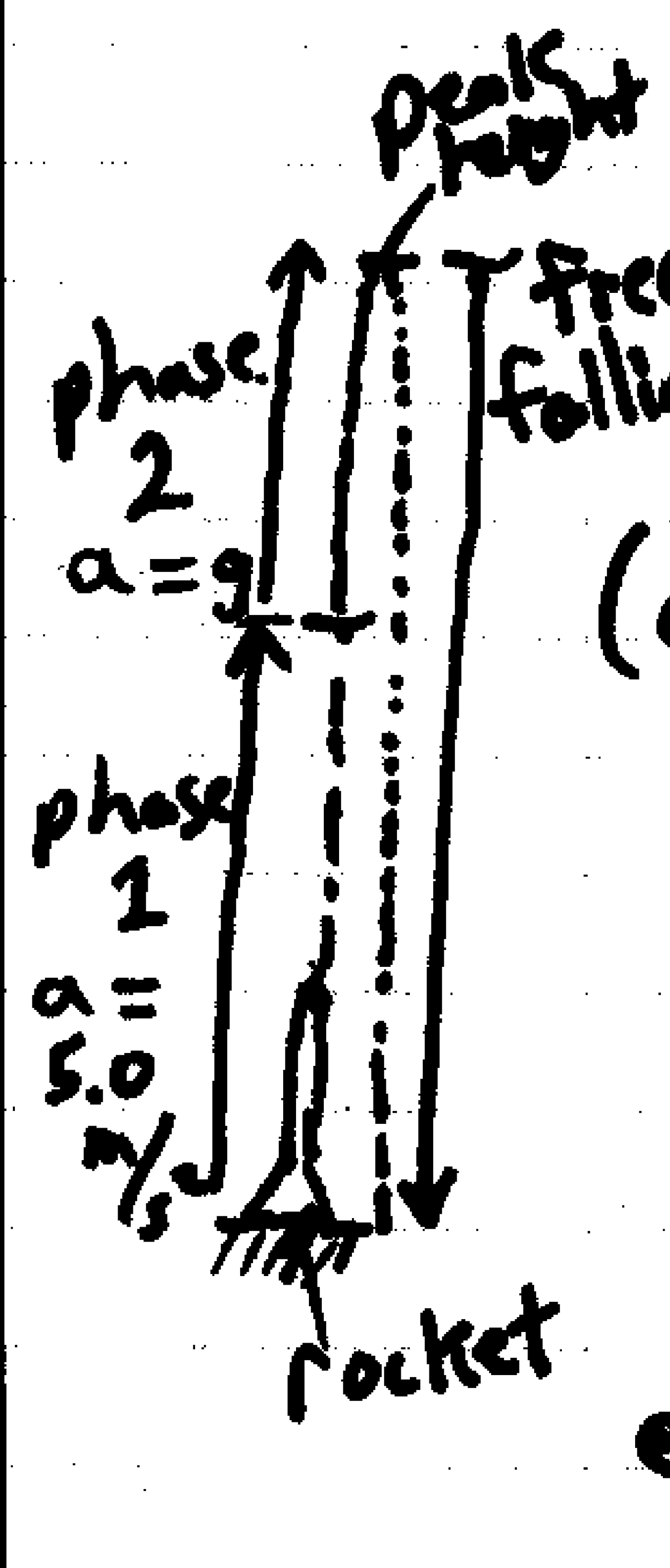
This is a 1-D problem involving free-fall acceleration.

A model rocket is launched vertically, and has a constant acceleration of 5.0 m/s^2 for 8.0 seconds, after which there is no fuel left.

(a) What is the maximum height reached by the rocket?

(b) How much time passes between when the rocket is launched and when it lands?

<u>Solution:</u>



Assumptions: - The rocket motion is purely vertical
- Air resistance is negligible

(a) There are two phases when the rocket is going up: A constant acceleration phase due to the rocket thrust, and a free-fall phase when only gravity is acting on the rocket.

In the first phase use the kinematic equation: $d_1 = v_1 t_1 + \frac{1}{2} a t_1^2$ $\uparrow +$ (sign convention)

$v_1 = 0$ (starts from rest)

$a = 5.0 \, m/s^2$ (due to rocket thrust)

$t_1 = 8.0 \, s$ (thrust duration)

$d_1 = ?$ (displacement during first phase)

Substitute:

$$d_1 = 0 + \frac{1}{2}(5.0)(8.0)^2$$

$$d_1 = 160\ m \text{ (first phase)}$$

In the second phase use the kinematic equation: $d_2 = v_2 t_2 + \frac{1}{2} g t_2^2$ (1)

$g = -9.8\ m/s^2$

$d_2 = ?$ (displacement during second phase)

$v_2 = ?$ (the speed at the start of the second phase, which is equal to the speed at the end of the first phase)

$t_2 = ?$ (duration of free-fall phase -when the rocket is going up) the second phase

Solve for v_2 using the kinematic equation: $v_2 = v_1 + at_1$

Substitute:

$$v_2 = 0 + (5.0)(8.0)$$

$$v_2 = 40\ m/s$$

Solve for t_2 using the kinematic equation: $v_3 = v_2 + g t_2$

$v_3 = 0$ (speed of rocket when maximum height is reached) at end of second phase

Substitute:

$$0 = 40 - (9.8)t_2$$

$t_2 = 4.08$ s (duration of free-fall phase)

From equation (1):

$$d_2 = v_2 t_2 + \frac{1}{2} g t_2^2$$

Substitute:

$$d_2 = (40)(4.08) - \frac{1}{2}(9.8)(4.08)^2$$

$d_2 = 81.6$ m (displacement during second phase)

The maximum height reached by the rocket is therefore $d_1 + d_2 = 160\text{ m} + 81.6\text{ m}$
$= 241.6$ m (answer)

(b) The rocket experiences free-fall between the point when maximum height is reached and when it lands. First find the time duration for this.

Use the kinematic equation: $d_3 = v_3 t + \frac{1}{2} g t^2$

$d_3 = -(d_1 + d_2) = -241.6$ m (negative because the rocket moves in the opposite direction when it falls)

$t = ?$ (falling time)

Substitute:

$$-241.6 = 0 - \frac{1}{2}(9.8)t^2$$

$t = 7.0$ s (Falling time)

Therefore, the total time between launch and landing is $t_1 + t_2 + t$

$= 8.0 + 4.08 + 7.0$

$= 19.1$ s (answer)