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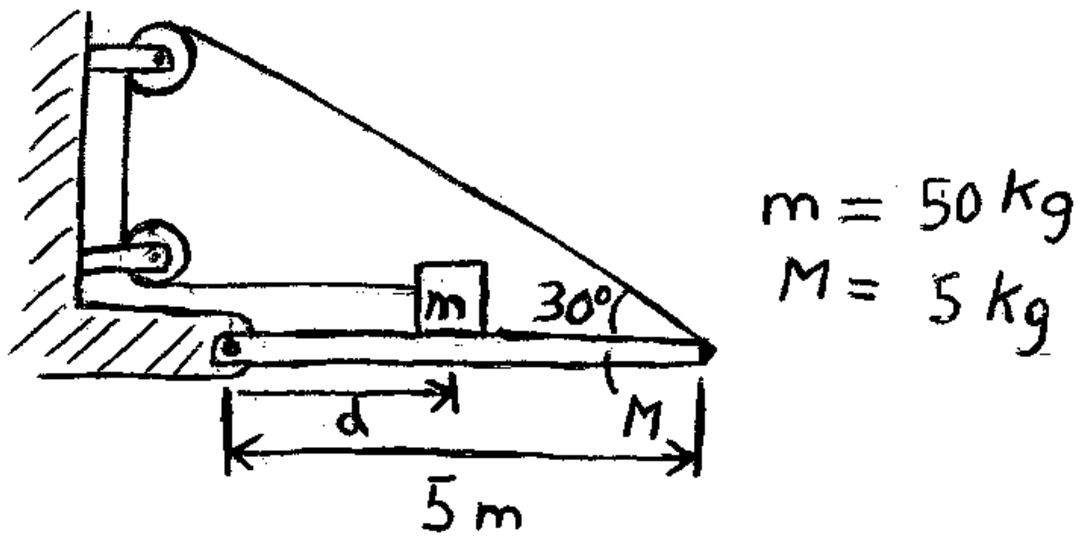
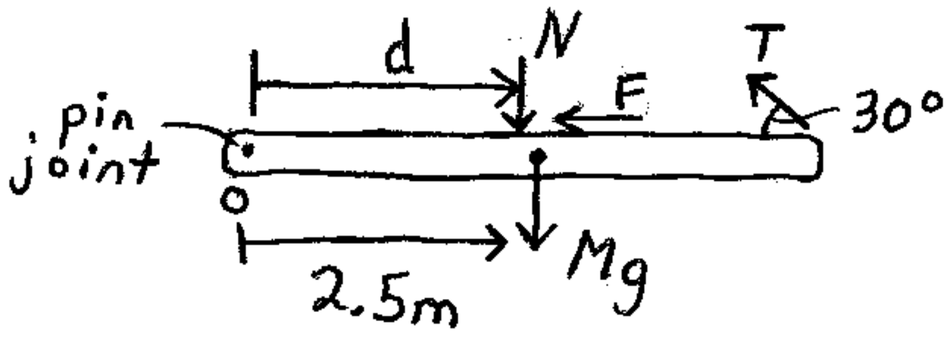
Nov. 12, 2020

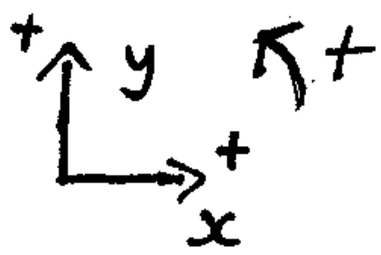
This is a problem involving statics.

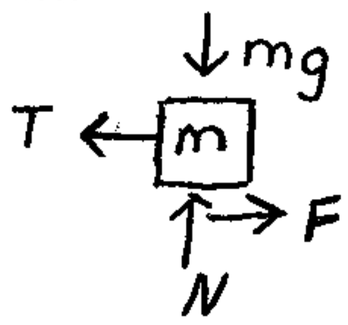


In the system shown, the coefficient of static friction between the mass m and the board M is 0.20. Determine the value of d so that the system is in equilibrium.

Solution:







For the block:

$F = \mu_s N$

$\Sigma F_x = 0 \Rightarrow F - T = 0 \Rightarrow F = T \quad (1)$

$\Sigma F_y = 0 \Rightarrow N - mg = 0 \Rightarrow N = mg \quad (2)$

For the board:

$\Sigma \tau_o = 0 \Rightarrow -Nd - Mg(2.5) + T\sin 30°(5) = 0 \quad (3)$

Note: F exerts negligible torque about O because the board has small thickness relative to its length.

Note: $\sum \tau_o = 0$ is the sum of the torque about the pivot O, which is 0 – and actually, the sum of the torque about any point is 0 for a static equilibrium problem.

substitute (1) and (2) into (3):

$$-mgd - Mg(2.5) + F\sin 30°(5) = 0$$

$$F = \frac{mgd}{2.5} + Mg \quad (4)$$

The maximum value of d occurs when F is the maximum possible value, which is when $F = \mu_s N$ (limiting static friction case)

So,

$$\text{Equation}(4) \Rightarrow \mu_s N = \frac{mgd}{2.5} + Mg$$

$N = mg$ from (2)

$$\Rightarrow \mu_s mg = \frac{mgd}{2.5} + Mg$$

$$\Rightarrow d = \frac{2.5}{mg}(\mu_s mg - Mg)$$

$$\Rightarrow d = \frac{2.5}{50(9.8)}(0.20(50)(9.8) - 5(9.8)) = 0.25m$$

This is the max. value of d.

$0 \leq d \leq 0.25m$ (for equilibrium)