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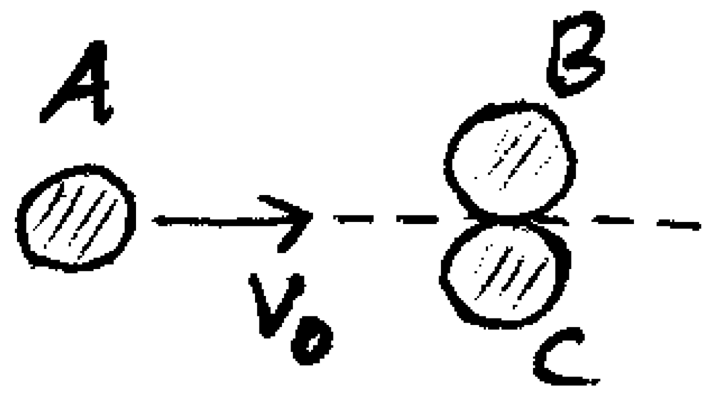
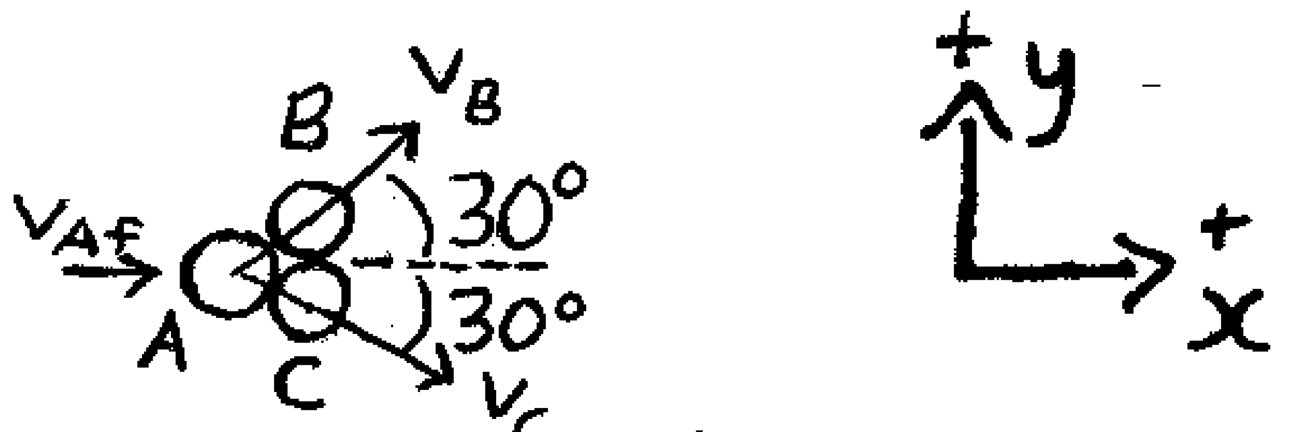
Nov. 12, 2020

This is a problem involving momentum.



Ball A is moving towards two balls, B and C, as shown, at a velocity of 5 m/s. Friction is negligible and the collision is elastic. If the balls are all identical, determine the velocities of all 3 balls after the collision.

Solution:



(Immediately after collision)

Linear momentum is conserved in the x and y direction. Ball A continues to move in x-direction after the collision.

$$mV_{Ai} = mV_{Af} + mV_B\cos 30° + mV_c\cos 30° \quad (x\text{-direction})$$

$$V_{Ai} = V_0, \quad \text{and } V_B = V_c = V, \text{ by symmetry}$$

$$\Rightarrow \quad V_0 = V_{Af} + 2V\cos 30° \quad (1)$$

and

$$0 = 0 + mV_B\sin 30° - mV_c\sin 30° \quad (y\text{-direction}) \Rightarrow 0 = V_B\sin 30° - V_c\sin 30°$$

so $V_B = V_c = V$, which is consistent with the symmetry argument that $V_B = V_c = V$

The collision is elastic so,

$$\frac{1}{2} m v_0^2 = \frac{1}{2} m v_{AF}^2 + \frac{1}{2} m v_B^2 + \frac{1}{2} m v_c^2$$

(kinetic energy is conserved)

$$v_B = v_c = v$$

$$\Rightarrow v_0^2 = v_{AF}^2 + 2v^2 \quad (2)$$

From (1), $v_{AF} = v_0 - 2v\cos 30°$

substitute this into (2):

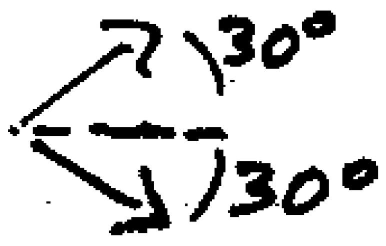
$$v_0^2 = v_0^2 - 4v_0 v\cos 30° + 4v^2\cos^2 30° + 2v^2$$

This simplifies to:

$$0 = -2v_0\cos 30° + 2v\cos^2 30° + v$$

$$\Rightarrow v = \frac{2v_0\cos 30°}{1+2\cos^2 30°}$$

(velocity of ball B and ball C after collision)



From (1):

$$v_{AF} = v_0 - 2\cos 30° \cdot \frac{2v_0\cos 30°}{1+2\cos^2 30°}$$

$$\Rightarrow v_{AF} = v_0 \frac{(1-2\cos^2 30°)}{(1+2\cos^2 30°)}$$

(velocity of ball A after collision)

$$v_0 = 5\ m/s$$