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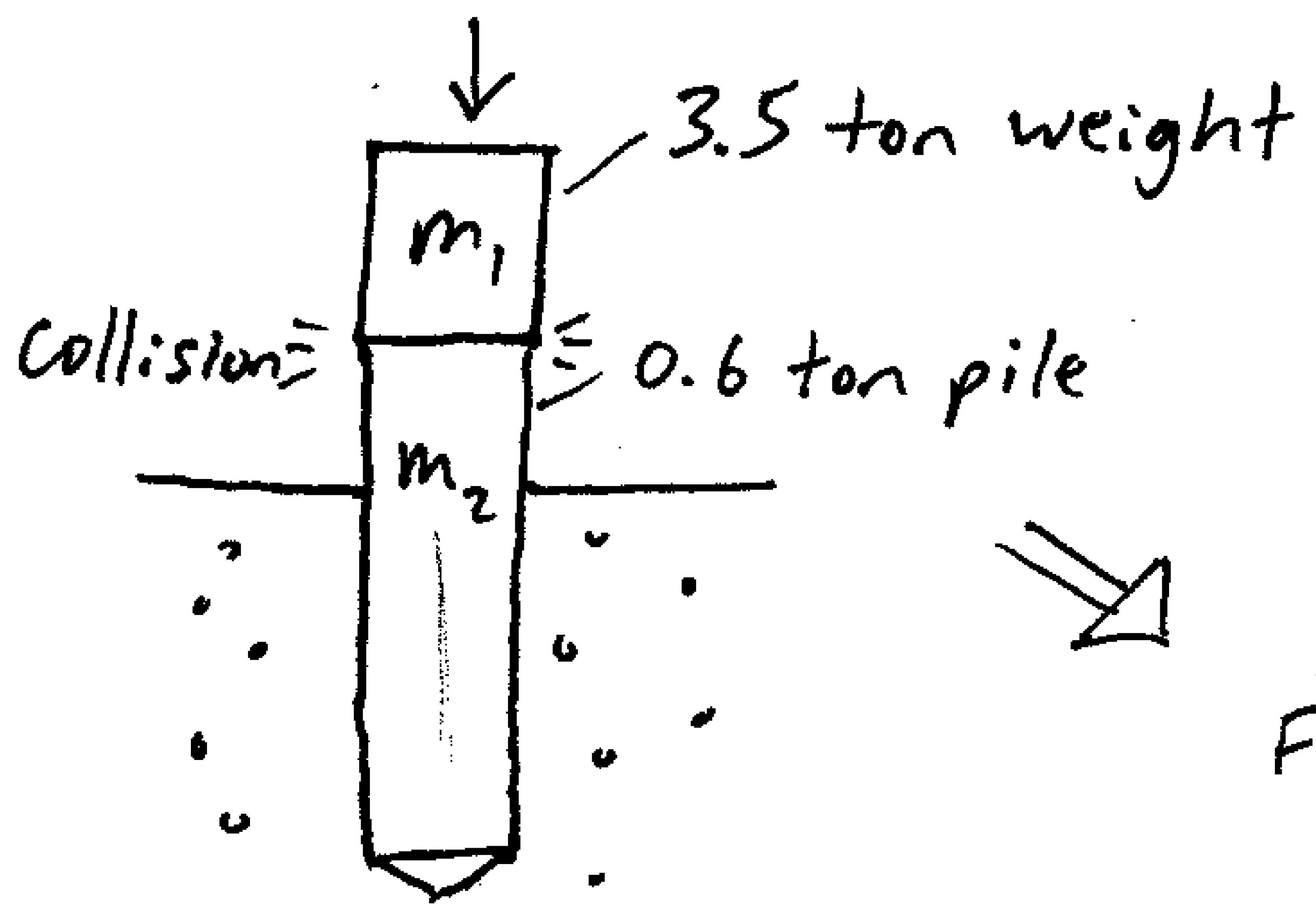
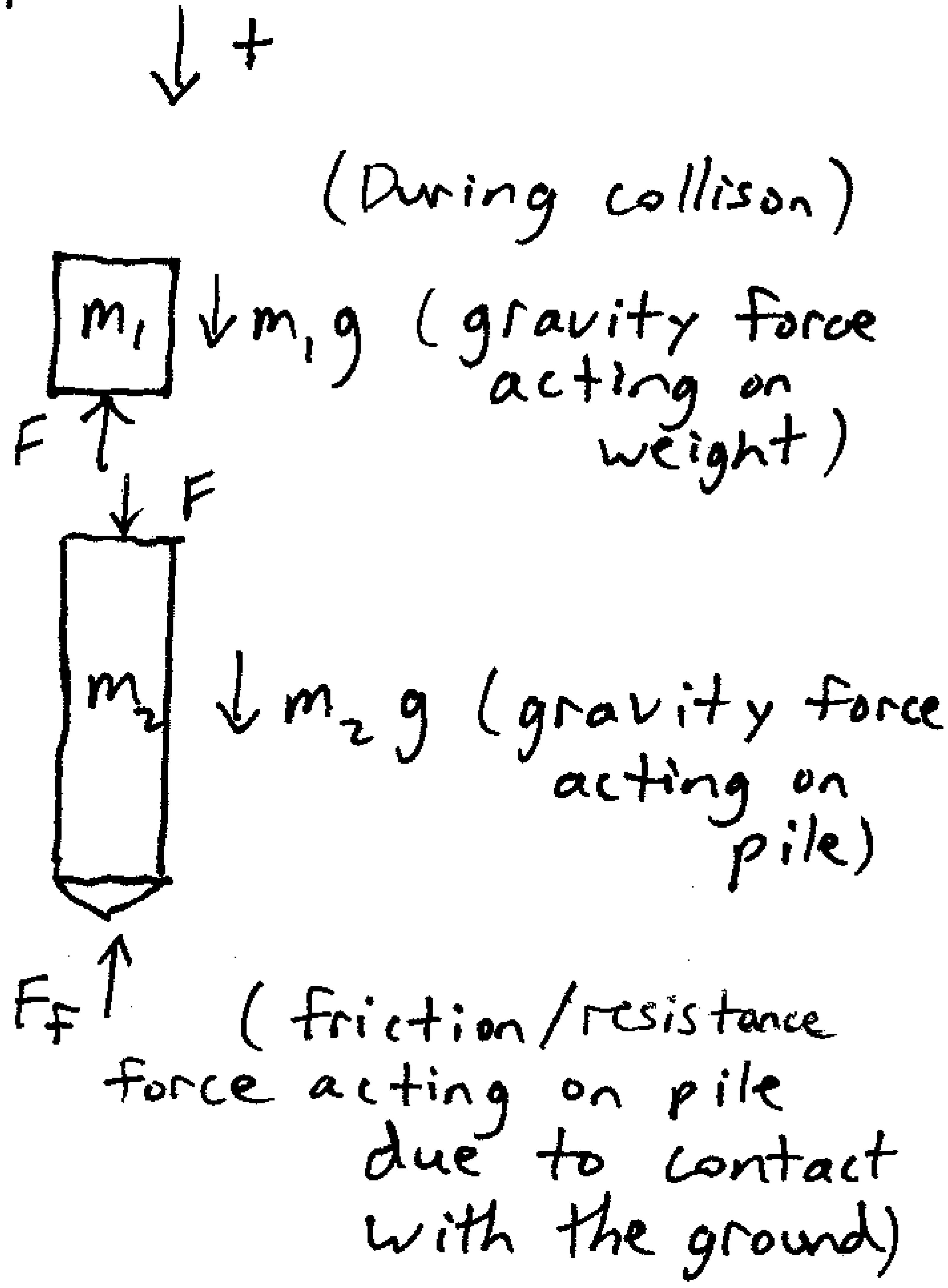
This is a problem involving momentum.

A 3.5 ton weight falls a distance of 1.8 m, and then impacts a 0.6 ton pile. As a result of this completely inelastic collision, the pile is driven 3 cm into the ground. Find the average force exerted on the pile by the ground.

Solution:





F is the impact force between the weight and pile during the collision

During the collision,
apply Newton's second law to the weight:

$$-F + m_1 g = m_1 a_1 \quad (1)$$

Apply Newton's second law to the pile, during the collision:

$$F + m_2 g - F_f = m_2 a_2 \quad (2)$$

During the collision, F is <u>much greater</u> than the gravity forces and the friction/resistance force. This means that

$$-F \approx m_1 a_1$$

and

$$F \approx m_2 a_2$$

which means that $-m_1 a_1 \approx m_2 a_2$,
and linear momentum is approximately conserved during impact.

$\Rightarrow$ The collision is completely inelastic, so

$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) V$, V is the velocity of weight + pile after impact.

Apply conservation of energy as m_1 falls 1.8 m, to determine v_{1i}:

$$\Rightarrow v_{1i} = \sqrt{2g(1.8m)} = \sqrt{3.6g}$$

$$\Rightarrow v_{2i} = 0$$

So, substitute:

$$m_1\sqrt{(3.6g)} = (m_1 + m_2)V$$

$$\Rightarrow V = \frac{m_1\sqrt{(3.6g)}}{m_1 + m_2}$$

Apply the principle of work and energy as the weight + pile is driven into the ground:

$$T_1 + \Sigma U_{1-2} = T_2$$

$$\Rightarrow \frac{1}{2}(m_1 + m_2)V^2 + \Sigma U_{1-2} = 0$$

$$\Sigma U_{1-2} = \underbrace{(m_1 + m_2)gL}_{\text{work done by gravity}} - \underbrace{\int F_f\, dL}_{\text{work done by friction/resistance, an integral since } F_f \text{ likely changes with time.}}$$

L is the distance the pile is driven into ground.

Replace this with $F_{avg}L$

Finally,

$$\frac{1}{2}(m_1 + m_2)V^2 + (m_1 + m_2)gL - F_{avg}L = 0$$

$$F_{avg} = (m_1 + m_2)\left(\frac{V^2}{2L} + g\right)$$

$$\Rightarrow F_{avg} = (m_1 + m_2)\left[\left(\frac{m_1}{m_1 + m_2}\right)^2 \frac{(3.6g)}{2L} + g\right]$$

For $m_1 = 3500\text{ kg}$, $m_2 = 600\text{ kg}$, $L = 0.03\text{ m}$, $F_{avg} = 1{,}800{,}000\text{ N}$ (answer)