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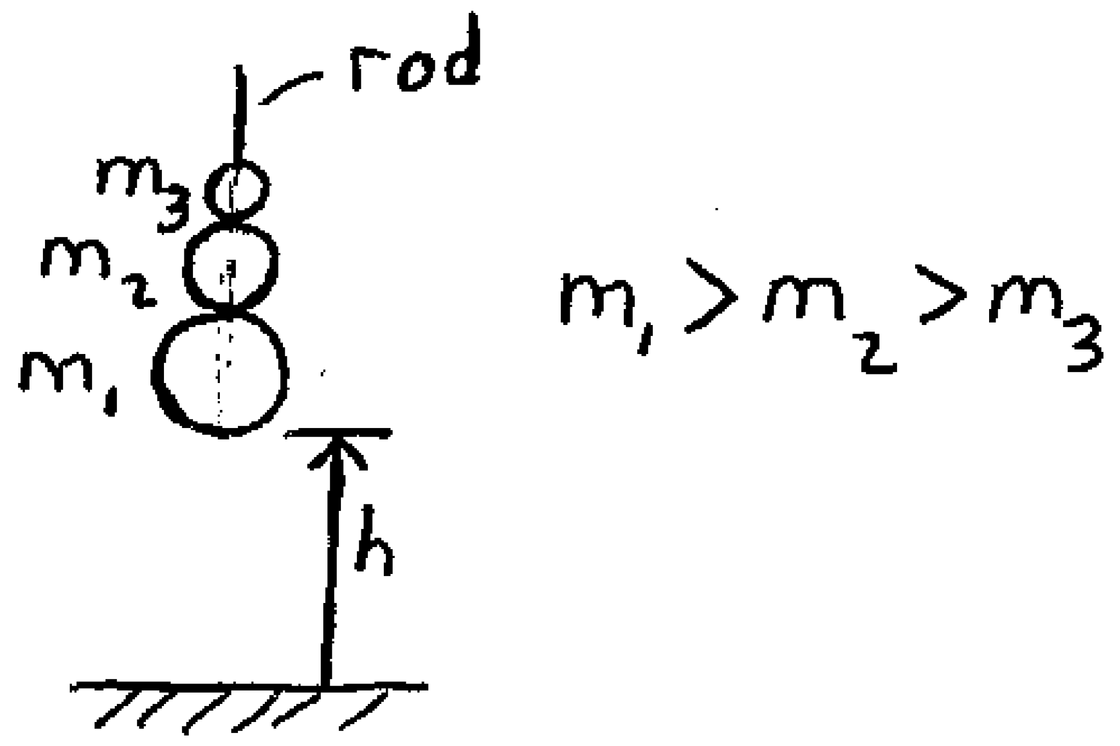
This is a problem involving momentum.



A toy consists of 3 rubber balls of mass m_1, m_2, and m_3, which have a thin rod threaded through them, allowing the balls to slide freely along the rod. The toy is dropped from a height h above the ground. If all collisions are elastic, what is the height reached by the ball with mass m_3?

Solution:

Treat this as an elastic collision problem in one-dimension, where all collisions are elastic.

Treat the problem as occuring in the following stages: $\uparrow +$

1. The balls fall a distance h, and all reach a velocity v downward just before impact with the ground.
2. The bottom ball (m_1) hits the ground first and rebounds with an upward velocity of v.
3. The bottom ball, moving at an upward velocity v, strikes the middle ball (m_2) which is still moving downward at velocity v.

4. The middle ball rebounds with an upward velocity v_1.

5. The middle ball, moving at an upward velocity v_1, strikes the top ball (m_3) which is still moving downward at velocity v.

6. The top ball rebounds with an upward velocity of v_2. From this, determine the height reached by the top ball.

First determine v_1 using information from steps 3 and 4, and apply 1-D elastic collision equation:

$$v_1 = \frac{2m_1}{m_1+m_2}(v) + \frac{(m_2-m_1)}{m_1+m_2}(-v)$$

$$v_1 = \left(\frac{3m_1 - m_2}{m_1+m_2}\right)v$$

Next, determine v_2 using information from steps 5 and 6, and apply 1-D elastic collision equation:

$$v_2 = \frac{2m_2}{m_2+m_3}(v_1) + \frac{(m_3-m_2)}{m_2+m_3}(-v)$$

Substitute $v_1 = \left(\frac{3m_1 - m_2}{m_1+m_2}\right)v$ into above equation:

$$v_2 = \frac{2m_2}{m_2+m_3}\left(\frac{3m_1-m_2}{m_1+m_2}\right)v + \frac{(m_3-m_2)(-v)}{m_2+m_3}$$

Simplify a bit:

$$V_2 = \frac{V}{m_2+m_3} \cdot \left(2m_2 \frac{(3m_1 - m_2)}{m_1+m_2} + m_2 - m_3 \right)$$

From conservation of energy, during the initial falling of the balls a distance h:

$$V = \sqrt{2gh}$$

From conservation of energy, during when the top ball flies upward after rebounding off the middle ball, the maximum height reached is:

$$H = \frac{V_2^2}{2g} \quad \text{(answer)}$$