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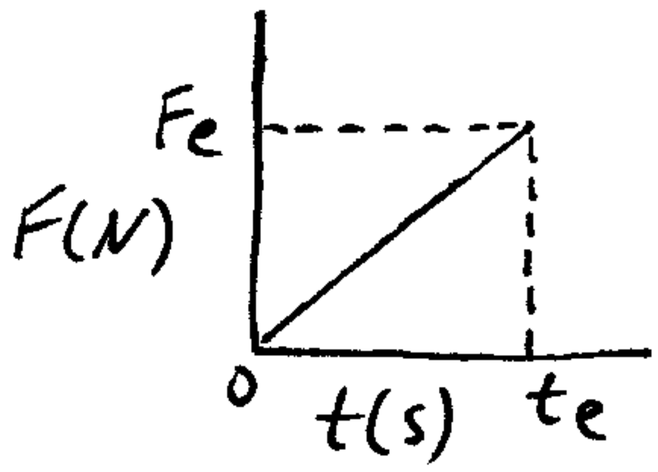
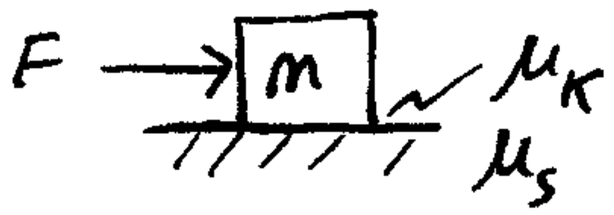
Nov. 12, 2020

This is a problem involving momentum.



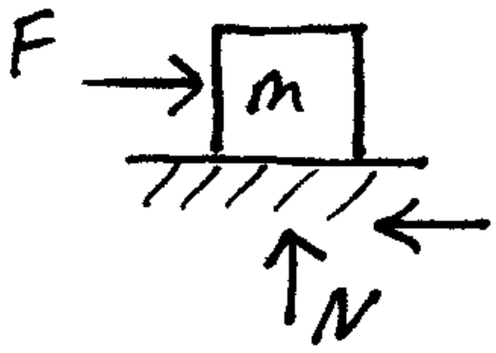


A block is initially at rest on a horizontal surface, and a force F is applied to it. Given the force vs. time graph, as shown, determine the velocity of the block at time t_e.

Assume $F_e > \mu_s mg$.

Solution:

(during sliding)



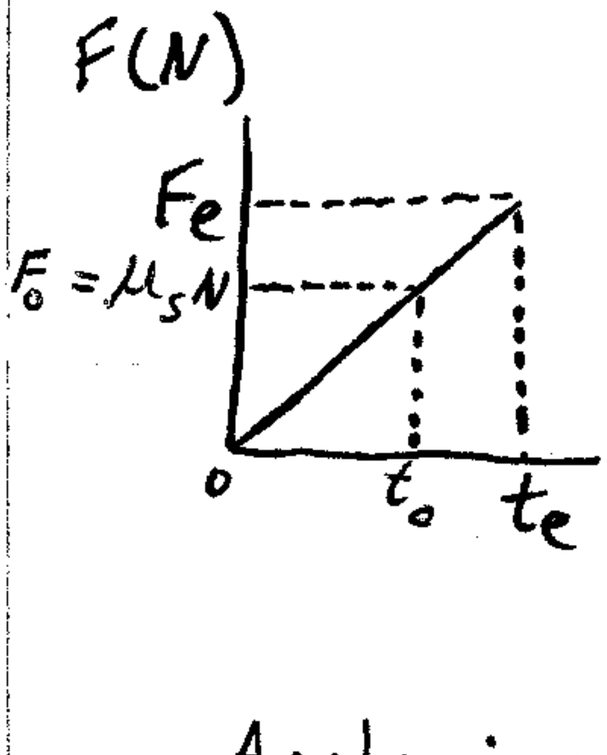
$\rightarrow +$

$N = mg$

$F_K = \mu_K N$ (Kinetic Friction) (during sliding)

(For sliding $F > \mu_s N$) (μ_s: static friction limit)

$(\mu_s > \mu_K)$



The block starts moving at time = t_0, when the static friction limit is exceeded.

Apply impulse and linear momentum, along direction of motion of block:

$$\int_{t_0}^{te} (\Sigma F_{ext}) \, dt = m v_f - m v_i \quad (1)$$

$$\Sigma F_{ext} = F - F_K \quad , \quad F_K = \mu_K N = \mu_K mg$$

$$F = \left(\frac{F_e}{t_e}\right) t$$

When $F = F_o = \mu_s N = \mu_s mg$

$$\Rightarrow \mu_s mg = \left(\frac{F_e}{t_e}\right) t$$

$$\Rightarrow t = t_o = \frac{\mu_s mg t_e}{F_e}$$

Therefore,

$$(1) \Rightarrow \int_{t_o}^{t_e} \left(\left(\frac{F_e}{t_e}\right) t - \mu_K mg\right) dt = m v_f - m v_i \quad (v_i \to 0 \text{ (starts from rest)})$$

$$\Rightarrow \frac{1}{2}\left(\frac{F_e}{t_e}\right) t^2 - \mu_K mg t \Big|_{t_o}^{t_e} = m v_f$$

$$\Rightarrow m v_f = \frac{1}{2}\left(\frac{F_e}{t_e}\right)\left(t_e^2 - t_o^2\right) - \mu_K mg\left(t_e - t_o\right)$$

$$\Rightarrow v_f = \frac{F_e t_e}{2m}\left(1 - \left(\frac{\mu_s mg}{F_e}\right)^2\right) - \mu_K g t_e\left(1 - \frac{\mu_s mg}{F_e}\right)$$

(answer)

$\mu_s > \mu_K$ and $t_e > t_o$

where $t_o = \dfrac{\mu_s mg t_e}{F_e}$