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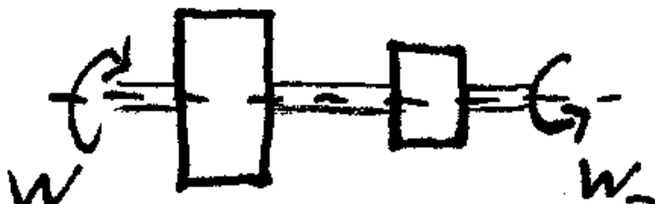
This is a problem involving momentum.



w_1 w_2 (angular velocity)
R_1 R_2 (radius)
m_1 m_2 (mass)

Two rotating cylinders are brought into contact, as shown. Due to friction between the cylinders, they eventually reach the same angular velocity. What is this angular velocity?

Note that the cylinders are initially rotating in opposite directions.

Solution:

Angular momentum is conserved, since the axis of rotation is the same for both cylinders, and the forces acting on each cylinder is equal and opposite to the forces acting on the other cylinder - which means that the torque acting on each cylinder (about the axis of rotation) is equal and opposite the torque acting on the other cylinder.

You have to assume there is no friction on the common axis of rotation of both cylinders (e.g. a well-lubricated shaft is used)

Therefore,

$$I_1 w_{1i} + I_2 w_{2i} = I_1 w_{1f} + I_2 w_{2f} \quad (1)$$

$$I_1 = \frac{1}{2} m_1 R_1^2 \quad , \quad I_2 = \frac{1}{2} m_2 R_2^2$$

$$w_{1i} = w_1 \quad , \quad w_{2i} = -w_2$$

$$w_{1f} = w_{2f} = w_f$$

Substitute above quantities into (1):

$$\Rightarrow \frac{1}{2} m_1 R_1^2 w_1 + \frac{1}{2} m_2 R_2^2 (-w_2) = \frac{1}{2} m_1 R_1^2 w_f + \frac{1}{2} m_2 R_2^2 w_f$$

$$w_f = \frac{m_1 R_1^2 w_1 - m_2 R_2^2 w_2}{m_1 R_1^2 + m_2 R_2^2} \qquad \text{(answer)}$$