

This is a problem involving systems of particles.

A 2-stage rocket has an initial mass of 1250 kg. At the end of its first stage, the rocket has a mass of 500 kg. At this point, the lower section of the rocket separates, and the top section of the rocket begins firing. The lower section of the rocket has a mass of 200 kg. At the end of the second stage, the mass of the rocket (consisting only of the top section) is 140 kg. For both stages, the rocket consumes fuel at the rate of 2.4 kg/s. The speed of the exhaust gases relative to the rocket engine is 2800 m/s. What is the speed increase of the rocket, and what is the thrust produced by the rocket? The rocket is in deep space where there is negligible gravitational force acting on it.

at end of second stage

Solution:

Use the rocket equation, for the first stage:

$$v_f - v_i = u \ln\left(\frac{M_i}{M_f}\right)$$

At the end of the first stage,

$$v_f = u \ln\left(\frac{M_i}{M_f}\right) = (2800 \text{ m/s}) \ln\left(\frac{1250 \text{ kg}}{500 \text{ kg}}\right) + v_i$$

$$v_f = 2565.6 + v_i \text{ m/s} \quad (\text{at end of first stage})$$

Similarly,
at the end of the second stage,

$$v_{2f} = u \ln\left(\frac{M_{2i}}{M_{2f}}\right) + v_{2i} \quad , \quad v_{2i} = v_{1f}$$

$$v_{2f} = (2800 \text{ m/s}) \ln\left(\frac{500 \text{ kg} - 200 \text{ kg}}{140 \text{ kg}}\right) + 2565.6 + v_{1i} \text{ m/s}$$

$$v_{2f} = 4700 + v_{1i} \text{ m/s}$$

The speed increase of the
rocket is:

$$v_{2f} - v_{1i} = 4700 \text{ m/s (answer)}$$

The thrust produced by
the rocket is:

$$T = (2.4 \text{ kg/s})(2800 \text{ m/s})$$

$$T = 6720 \text{ N (answer)}$$