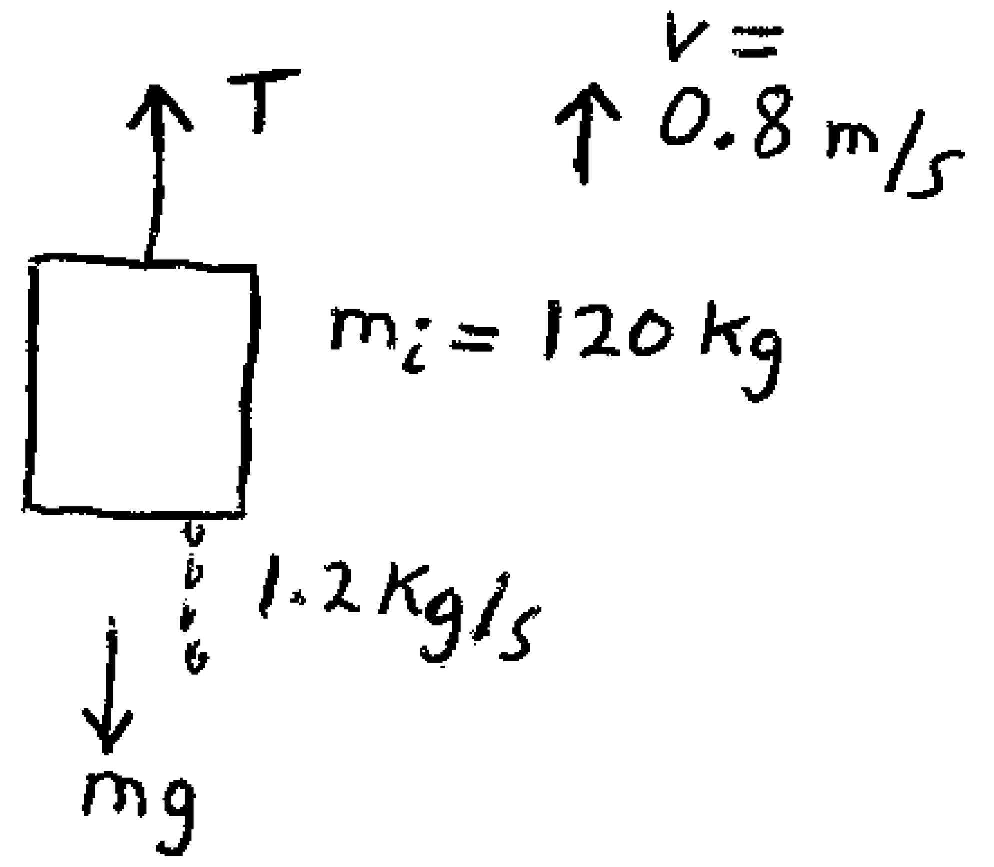
Nov. 9, 2020

This is a problem involving systems of particles.

A water container of initial mass 120 kg is being lifted at constant velocity, $v = 0.8\ m/s$. The container has a leak and is losing 1.2 kg of water per second. Find an expression for the force required to lift the container, over time.

What if the container was instead accelerating upward at $0.4\ m/s^2$?

Solution:



$\uparrow +$ (y-direction)

For a system that is losing mass:

$$\underbrace{\Sigma F_y}_{\text{sum of external forces in y-direction}} = m\frac{dv}{dt} - u\frac{dm_e}{dt} \quad (1)$$

m = mass of water container, which changes with time

v = velocity of container

u = relative velocity of container as seen by an observer moving with the ejected water

$\frac{dv}{dt}$ = acceleration of container

$\frac{dm_e}{dt}$ = rate at which water is leaving container

T = force required to lift container

$$\Sigma F_y = T - mg$$

Substitute this into equation (1):

$$T - mg = m\frac{dv}{dt} - u\frac{dm_e}{dt} \quad (2)$$

Now, $m = 120 - 1.2t$ kg, t = time in seconds

$v = 0.8\,m/s$ (constant)

$u = 0$ (since the water dripping from the container is always moving at the same velocity as the container, at the instant the drops fall out)

$$\frac{dv}{dt} = 0$$

$$\frac{dm_e}{dt} = 1.2\,kg/s$$

Substitute the above quantities into equation (2):

$$\Rightarrow T - (120 - 1.2t)g = (120 - 1.2t)(0) - (0)(1.2)$$

$$\Rightarrow T = (120 - 1.2t)g$$ (answer for the first part of the question)

For the second part of the question, $\frac{dv}{dt} = 0.4\,m/s^2$, and substitute this into equation (2), keeping all other quantities the same as before:

$$\Rightarrow T - (120 - 1.2t)g = (120 - 1.2t)(0.4) - (0)(1.2)$$

$$\Rightarrow T = (120 - 1.2t)(g + 0.4)$$ (answer for second part of the question)

Reminder, $g = 9.8\,m/s^2$