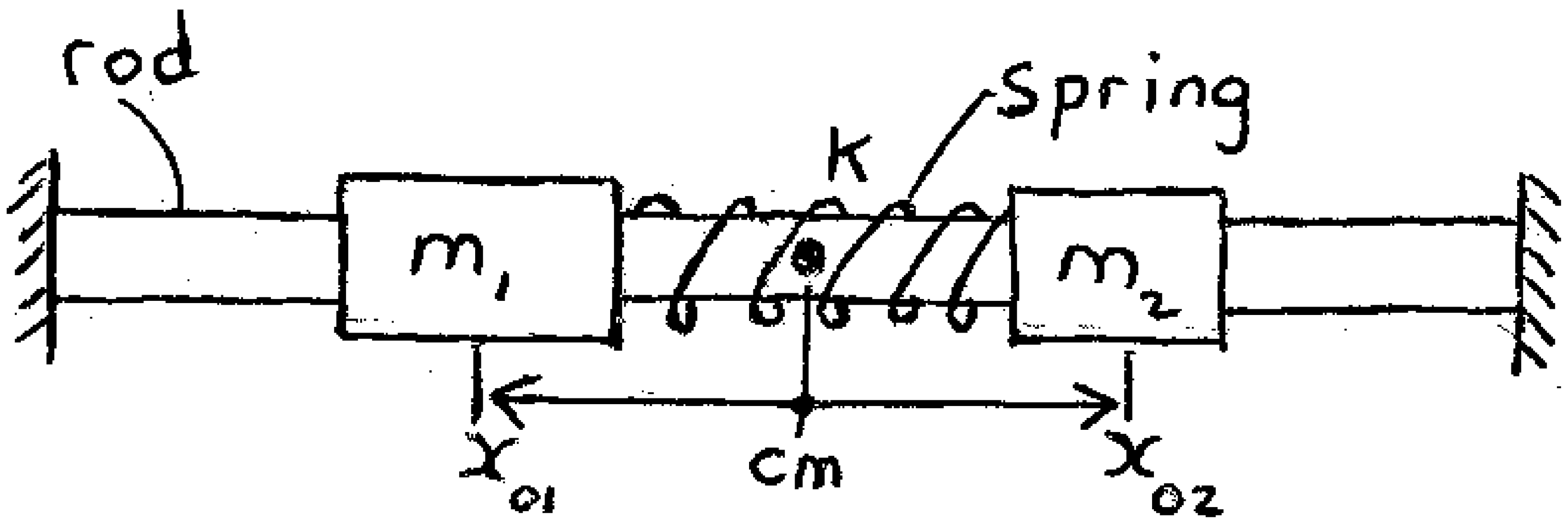
real-world-physics-problems.com
Nov. 8, 2020

This is a problem involving systems of particles.

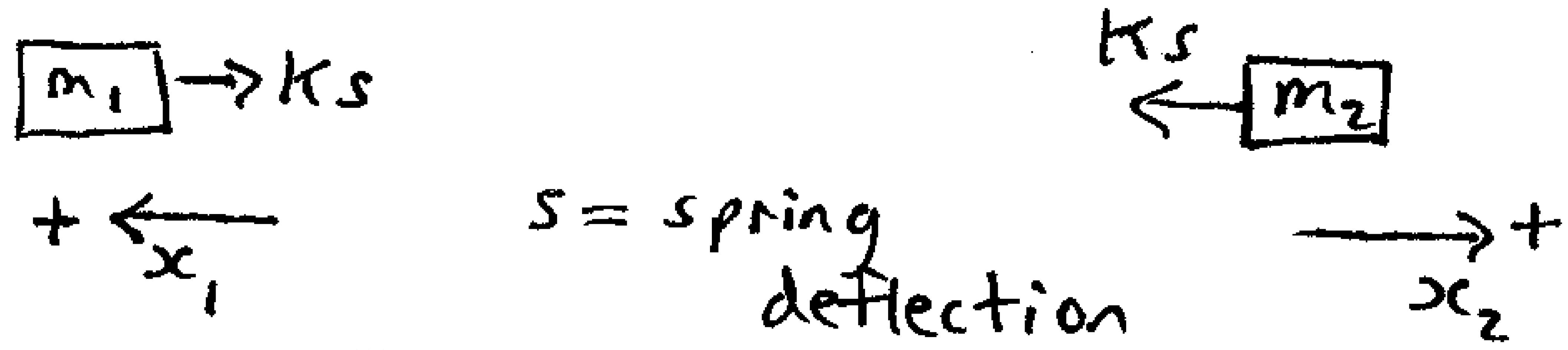


Two masses, m_1 and m_2, can slide without friction on a horizontal rod. A spring of stiffness k is connected between the two masses. At the instant shown, the masses m_1 and m_2 are located a distance x_{01} and x_{02} from the center of mass (cm) of the system, respectively. At this instant the spring is stretched by an amount ΔL and the velocity of the two masses is zero. Find expressions for the distances x_1 and x_2 over time, when system released. Note that the system is defined here as the masses m_1 and m_2, and the spring. The mass of the spring is small relative to m_1 and m_2, so it can be ignored.

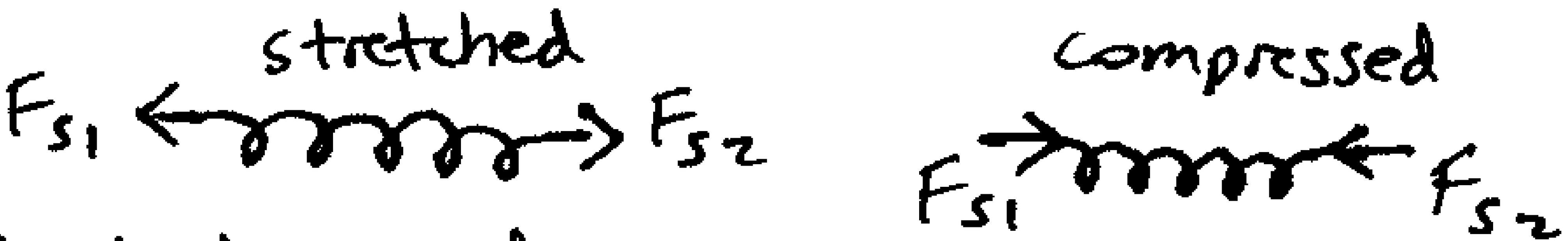
<u>Solution</u>:

There are no net external forces, in horizontal direction, acting on the system, so the position of cm (center of mass) is constant, in the horizontal direction. There are no net external forces acting in the vertical direction either, but you don't need to consider the vertical direction.

Free-body diagram



s = spring deflection



Apply Newton's second law to spring: $\sum F_{spring} = ma_{cm}$

m = mass of spring
a_{cm} = acceleration of spring center of mass
$\Rightarrow m \approx 0$
$\Rightarrow \sum F_{spring} \approx 0$
so, $F_{s1} = F_{s2} = Ks$

$$(1) \quad s = (x_1 - x_{01}) + (x_2 - x_{02}) + \Delta L$$

L_0 = spring length when spring is neither stretched nor compressed

$L = L_0 + s$

L: spring length

$s > 0$ (spring is stretched)
$s < 0$ (spring is compressed)

Apply Newton's second law to the mass m_1:

$$-Ks = m_1 \frac{d^2 x_1}{dt^2} \qquad (2)$$

Apply Newton's second law to the mass m_2:

$$-Ks = m_2 \frac{d^2 x_2}{dt^2} \qquad (3)$$

Substitute (1) into (2) and (3).

The initial conditions are, at time $= 0$:

$$x_1(0) = x_{01}$$
$$x_2(0) = x_{02}$$
$$\left.\begin{array}{l} \dfrac{dx_1(0)}{dt} = 0 \\[2ex] \dfrac{dx_2(0)}{dt} = 0 \end{array}\right\} \text{initial velocity of masses is zero}$$

The known form of the solution is:

$$x_1(t) = A_1 \cos(w_1 t) + B_1 \cos(w_2 t) + C_1 \qquad (4)$$

$$x_2(t) = A_2 \cos(w_1 t) + B_2 \cos(w_2 t) + C_2 \qquad (5)$$

where:
$w_1, w_2, A_1, A_2, B_1, B_2, C_1, C_2$ are constants

Substitute (4) and (5) into (2) and (3) and solve for the above constants. The algebra for this is not shown here, but it's a useful exercise to go through.

The result is:

$$x_1(t) = \left(\frac{\Delta L m_2}{m_1 + m_2}\right)\cos(wt) + x_{01} - \frac{\Delta L m_2}{m_1 + m_2}$$

$$x_2(t) = \left(\frac{\Delta L m_1}{m_1 + m_2}\right)\cos(wt) + x_{02} - \frac{\Delta L m_1}{m_1 + m_2}$$

where $w = \sqrt{\frac{K(m_1 + m_2)}{m_1 m_2}}$