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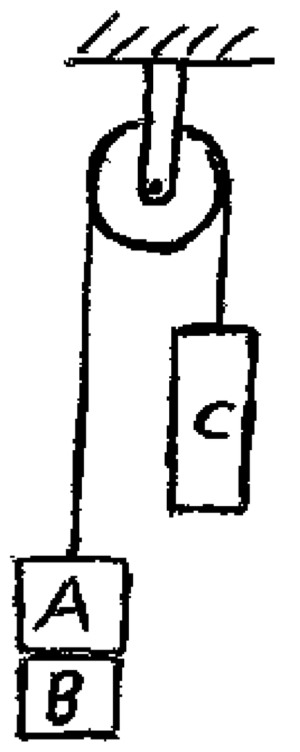
This is a problem involving conservation of energy.



In the pulley system shown, $m_A = 4$ kg, $m_B = 5$ kg, and $m_C = 11$ kg. The masses A and B are glued to each other. The system is released from rest and when mass C has fallen 0.4 m, mass B detaches from mass A due to poor quality glue. Determine the velocity of mass C when it has fallen 0.7 m. Ignore the mass of the rope and pulley.

Solution:

Apply conservation of energy:

$$T_1 + V_1 = T_2 + V_2 \qquad (1)$$

$\downarrow g$ $\uparrow +$ $\downarrow -$

In the first stage, the masses A and B remain joined to each other.

In this stage:

start of first stage:
$T_1 = 0$ (system starts from rest)
$V_1 = 0$ (relative to two datums placed at the center of mass of combined mass A and B, and mass C)

end of first stage

$$T_2 = \frac{1}{2}(m_A + m_B)v^2 + \frac{1}{2} m_C v^2$$

Kinetic energy of combined mass A and B, moving at velocity v, upward.

Kinetic energy of mass C, moving at velocity v, downward.

$$V_2 = (m_A + m_B)g(0.4) + m_C g(-0.4)$$

potential energy of combined mass A and B, after it has moved a positive displacement upward relative to its datum.

potential energy of mass C, after it has moved a negative displacement downward relative to its datum.

substitute the above quantities into equation (1), along with given masses:

$$0 = \frac{1}{2}(9)v^2 + \frac{1}{2}(11)v^2 + 9(9.8)(0.4) - 11(9.8)(0.4)$$

solve for $v = 0.885$ m/s (velocity at end of first stage)

In the second stage, mass B is no longer present.

In this stage:

start of second stage:

$$T_1 = \frac{1}{2} m_A v^2 + \frac{1}{2} m_c v^2$$

$V_1 = 0$ (relative to two datums placed at the center of mass of mass A and mass C)

end of second stage:

$$T_2 = \frac{1}{2} m_A v_F^2 + \frac{1}{2} m_c v_F^2$$

$$V_2 = m_A g(0.3) + m_c g(-0.3)$$

$m_A g(0.3)$: positive displacement upward relative to its datum, where $\Delta d = 0.7m - 0.4m = 0.3m$

$m_c g(-0.3)$: negative displacement downward relative to its datum, where $\Delta d = -(0.7m - 0.4m) = -0.3m$

Substitute the above quantities into equation (1), along with given masses:

$$\frac{1}{2}(4)(0.885)^2 + \frac{1}{2}(11)(0.885)^2 + 0$$
$$= \frac{1}{2}(4)v_F^2 + \frac{1}{2}(11)v_F^2 + 4(9.8)(0.3) - 11(9.8)(0.3)$$

solve for $V_F = 1.88$ m/s (answer)
(downward for mass C)