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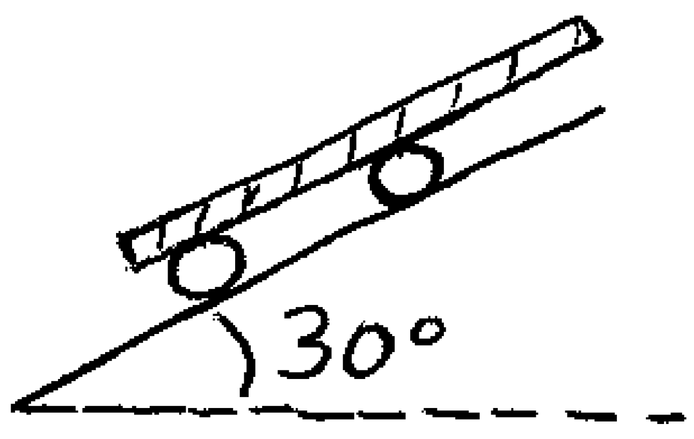
This is a problem involving conservation of energy.



A board is supported by two cylinders on an incline, as shown. The mass of the board is 12 Kg, the mass of each cylinder is 5 kg and their radius is 5 cm. If the system is released from rest and there is no slipping anywhere, what is the velocity of the board after it has moved a distance of 1.2 meters? Assume the board stays in contact with the cylinders.

Solution:

Apply the conservation of energy:

$$T_1 + V_1 = T_2 + V_2 \quad (1)$$

$T_1 = 0$ (initial kinetic energy, given that the system starts from rest)

$V_1 = 0$ (initial gravitational potential energy of system, relative to three datums located at the center of mass of the board, and the center of mass of each cylinder)

$$T_2 = \underbrace{2\left(\frac{1}{2}m_c v_G^2 + \frac{1}{2} I_G w^2\right)}_{\text{Final Kinetic energy of each cylinder}} + \underbrace{\frac{1}{2}m_b v_b^2}_{\text{Final Kinetic energy of board}}$$

where:

m_c = mass of each cylinder

V_G = velocity of center of mass of each cylinder

I_G = rotational inertia of each cylinder about center of mass

w = angular velocity of each cylinder

m_b = mass of board

V_b = velocity of board

Due to the Kinematics, $V_b = 2V_G$ (2) since there is no slipping between board and cylinder

The cylinders roll without slipping on incline, so $V_G = wR$ (3)

Next, R = radius of each cylinder

$$V_2 = -m_c g\left(\frac{L\sin 30°}{2}\right)\times 2 - m_b g\,(L\sin 30°)$$

where:

L = the distance that the board moves, which is 1.2 m

Note that the change in height of the board is $L\sin 30°$. Due to the Kinematics, the center of mass of each cylinder moves a distance $\frac{L}{2}$, which is a consequence of equation (2), and this means that the change in height of each cylinder is $-\frac{L}{2}\sin 30°$.

Substitute the different quantities into equation (1):

$$0 = 2\left(\frac{1}{2}m_c V_G^2 + \frac{1}{2}I_G w^2\right) + \frac{1}{2}m_b V_b^2$$
$$- m_c g\left(\frac{L\sin 30°}{2}\right)\times 2 - m_b g\,(L\sin 30°)$$

From equation (3) substitute $w = \frac{V_G}{R}$, $I_G = \frac{1}{2}m_c R^2$, and $V_b = 2V_G$ from equation (2) and simplify.

The above equation simplifies to:

$$0 = m_c V_G^2 + \frac{1}{2} m_c V_G^2 + 2 m_b V_G^2 - m_c g \frac{L}{2} - m_b g \frac{L}{2}$$

Solve:

$$V_G = \sqrt{\frac{g \frac{L}{2} (m_b + m_c)}{2m_b + \frac{3}{2} m_c}}$$

substitute $m_b = 12\,Kg$, $m_c = 5\,Kg$, $g = 9.8\,m/s^2$, $L = 1.2\,m$

$V_G = 1.78\ m/s$

From equation (2), $V_b = 2V_G = 3.56\ m/s$ (ans.)