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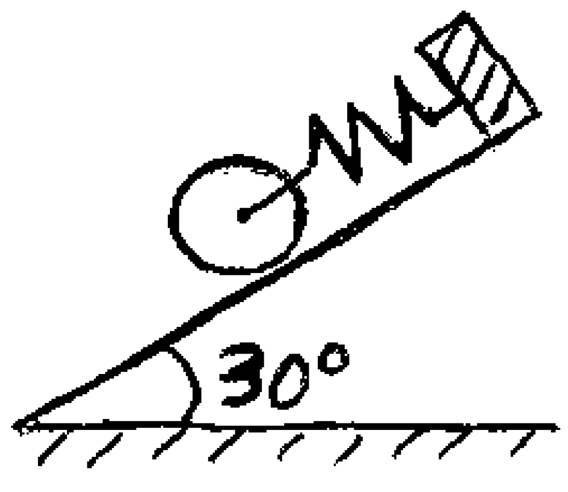
This is a problem involving conservation of energy.



A cylinder is attached to a spring, while on an incline, as shown. The cylinder is released from rest when the spring is neither stretched nor compressed. The mass of the cylinder is 25 kg, its radius is 8 cm, and the spring stiffness is 500 N/m. What is the maximum and minimum speed of the cylinder, and the corresponding stretch amount of the spring? Assume the cylinder rolls without slipping.

Solution:

Put the datum at the initial location of the center of mass of the cylinder. The cylinder then moves a distance L down the incline.

Apply the conservation of energy:

$$T_1 + V_1 = T_2 + V_2 \quad (1)$$

$T_1 = 0$ (initial kinetic energy, starts from rest)

$V_1 = 0$ (initial gravitational potential energy of cylinder, relative to datum (is zero), plus spring potential energy - unstretched - so its potential energy is also zero)

$$T_2 = \underbrace{\frac{1}{2} m v_G^2}_{\text{translational component of kinetic energy}} + \underbrace{\frac{1}{2} I_G w^2}_{\text{rotational component of kinetic energy}}$$

(final kinetic energy of cylinder - the mass of the spring is assumed negligible so its kinetic energy is also negligible)

For the cylinder rolling without slipping, $V_G = wR$, $R = 0.08\,m$

$$\Rightarrow w = \frac{V_G}{R}$$

$$\Rightarrow T_2 = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \left(\frac{V_G}{R}\right)^2$$

$$V_2 = -\overset{(mg)x}{L}\sin 30° + \frac{1}{2} K L^2$$

(final gravitational potential energy of cylinder, relative to datum, plus final spring potential energy)

Since the cylinder is rolling without slipping, there is no frictional energy loss, unlike, say, a block sliding down an incline, which loses energy due to kinetic friction. In this situation energy is not conserved.

Substitute known quantities into equation (1):

$$0 = \frac{1}{2} m V_G^2 + \frac{1}{2} I_G \left(\frac{V_G}{R}\right)^2 - \overset{(mg)x}{L}\sin 30° + \frac{1}{2} K L^2$$

The minimum speed of the cylinder is $V_G = 0$ and occurs when:

$$0 = -mgL\sin 30° + \frac{1}{2} K L^2$$

$m = 25\,kg$, $g = 9.8\,m/s^2$, $K = 500\,N/m$

Solve for L: $L = 0$ or $L = 0.49\,m$ (stretch amount of spring)

(answer)

For the maximum speed, maximize v_G in the above equation, which is the same as maximizing v_G^2. First, rewrite the above equation:

$$v_G^2\left(\frac{1}{2}m + \frac{1}{2}\cdot\frac{I_G}{R^2}\right) = mgL\sin 30° - \frac{1}{2}kL^2$$

$$I_G = \frac{1}{2}mR^2$$ (rotational inertia of cylinder about its cater of mass)

substitute I_G:

$$\Rightarrow v_G^2\left(\frac{1}{2}m + \frac{1}{4}m\right) = mgL\sin 30° - \frac{1}{2}kL^2$$

Now, $m = 25\text{ kg}$, $g = 9.8\text{ m/s}^2$, $k = 500\text{ N/m}$

substitute above:

$$\Rightarrow v_G^2(18.75) = \underbrace{122.5L - 250L^2}$$

This is a parabolic equation with maximum value of 15.0 at $L = 0.245\text{ m}$

So, $$v_G^2 = \frac{15.0}{18.75}$$

$$\Rightarrow v_G = 0.895\text{ m/s}$$ (maximum speed of cylinder which occurs at a stretch amount of $L = 0.245\text{ m}$)

(answer)