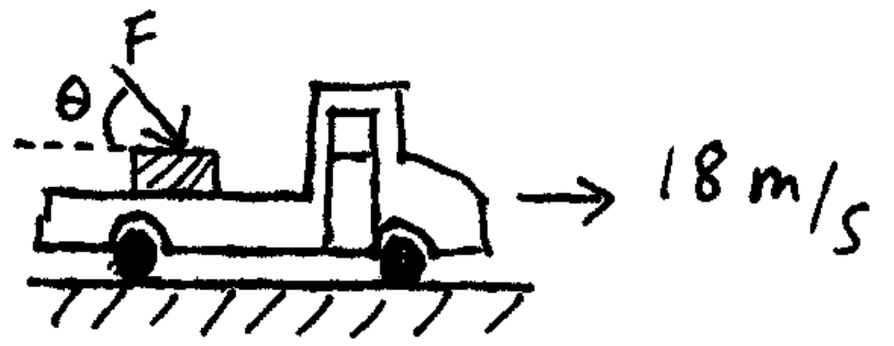
Nov. 5, 2020

This is a problem involving work and energy.



A block on the back of a truck, traveling at a constant velocity of 18 m/s, has a force F with magnitude 120 N applied to it. The coefficient of kinetic friction between the block and truck surface is 0.15. What is the velocity of the block when it has moved a distance of 2.5 m, relative to the truck? Calculate the answer from the point of view of an observer on the ground, and on the truck. The mass of the block is $m = 3\,kg$, and $\theta = 30°$. The block is initially at rest on the truck.

Solution:

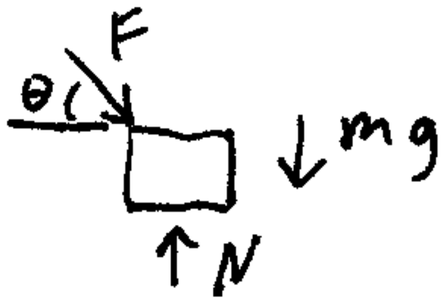
Apply the principle of work and energy:

$$T_1 + \Sigma U_{1-2} = T_2$$

Since the truck is moving at constant velocity it can serve as an inertial reference frame for which the above equation applies. According to this reference frame (which is the reference frame of an observer on the truck): (work done by friction)

$$\frac{1}{2}mv_1^2 + \underbrace{F\cos\theta\,(2.5)}_{\text{work done by force}} - \overbrace{\mu_K N\,(2.5)} = \frac{1}{2}mv_2^2$$

$v_1 = 0$ (starts from rest on truck)



$N = F\sin\theta + mg$ (no vertical acceleration)

$N = 120\sin 30° + (3)(9.8) = 89.4\ N$

Substitute known values into above equation:

$$120\cos 30°\,(2.5) - (0.15)(89.4)(2.5) = \frac{1}{2}(3)v_2^2$$

$v_2 = 12.3\ m/s$, from the point of view of an observer on the truck. (answer)

this is an inertial reference frame

Now, use the ground reference frame. First determine the distance that the truck moves during the time interval that the block slides on the truck.

Apply Newton's second law, using the reference frame of the truck:

$$F\cos 30° - \mu_K N = ma$$
$$\Rightarrow 120\cos 30° - 0.15(89.4) = 3a$$
$$\Rightarrow a = 30.171\ m/s^2$$

Next, apply the kinematics equation to find the time interval, Δt, again in truck reference frame

$$\Delta d = v_i \Delta t + \frac{1}{2}a\Delta t^2$$
($v_i \to 0$)

For $\Delta d = 2.5\ m$, $\Delta t = 0.4071\ s$

In this time interval, the truck moves a distance, $D = (18\ m/s)(0.4071\ s) = 7.328\ m$

Lastly, apply the work and energy equation:

$$\frac{1}{2}(3)(18)^2 + 120\cos 30°(D+2.5) - 0.15(89.4)(D+2.5) = \frac{1}{2}(3)v_2^2$$

(ans.)

initial velocity of block is equal to truck velocity

block moves truck distance plus 2.5m

For $D = 7.328\ m$, $v_2 = 30.3\ m/s$ which is consistent with truck frame where $v_2 = 18\ m/s + 12.3\ m/s$