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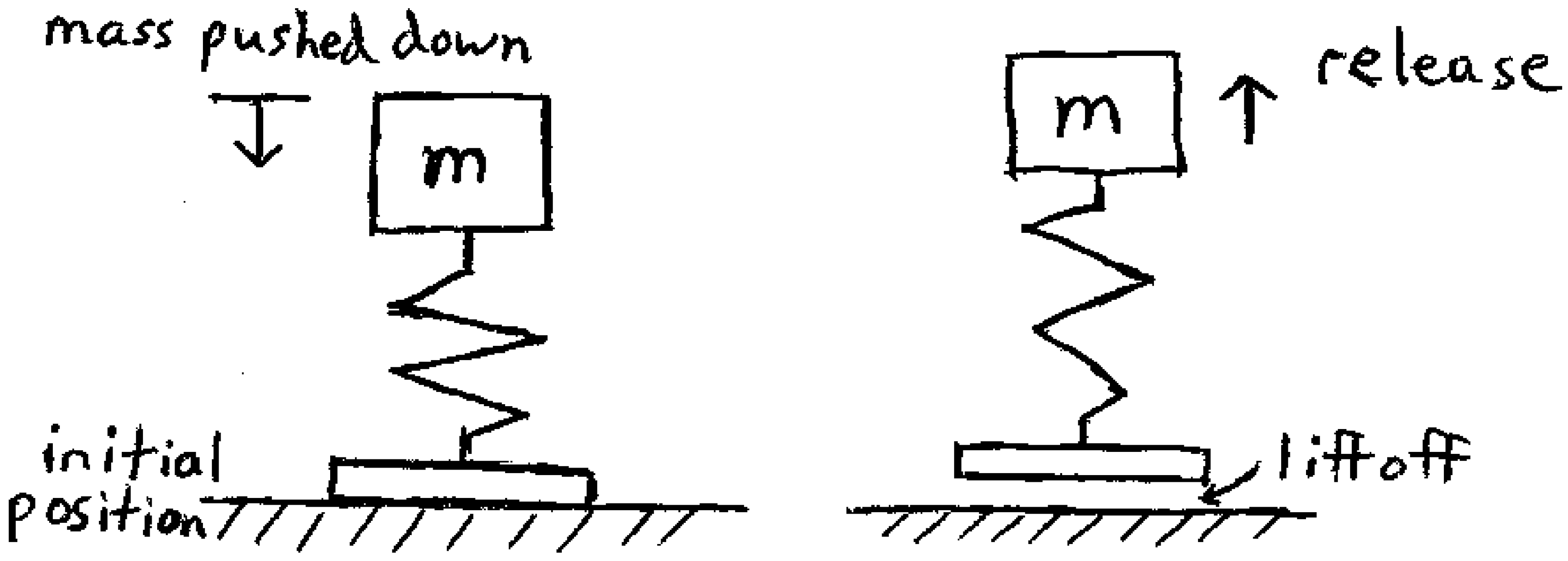
Nov. 1, 2020

This is a problem involving work and energy.



A mass m is attached to a spring, and pushed downward so that the spring is compressed 30 cm. The mass is then released and the spring-mass system lifts off from the ground, as shown. The maximum liftoff height is 0.60 m. If the mass $m = 15\,kg$, what would be the maximum liftoff height on the moon, given that the acceleration due to gravity, on the moon, is one-sixth that on Earth? Ignore the mass of the spring and bottom support for spring.

Solution:

Apply the principle of work and energy, from when the spring is initially compressed to when maximum liftoff height is reached:

$$T_1 + \Sigma U_{1-2} = T_2 \qquad (1)$$

$$\Sigma U_{1-2} = \underbrace{-\left(\tfrac{1}{2}ks_2^2 - \tfrac{1}{2}ks_1^2\right)}_{\text{Work done by spring}} + \underbrace{W\Delta d}_{\text{Work done by gravity}} \qquad (2)$$

Work done by spring

K is unknown

Work done by gravity. This is <0 because gravity and displacement (Δd) of mass are in opposite directions.

$\uparrow +$

$W = -mg = -(15)(9.8)$, $\Delta d = s_1 + 0.60\ m = 0.90\ m$

(gravity acts down) - in the negative direction

$s_1 = 0.30m$ (initial spring compression, when mass is released)
$s_2 = 0$ (at lift off, and after lift off, when maximum height is reached)

$T_1 = 0$ (mass is released from rest)

$T_2 = 0$ (when maximum height is reached)

Therefore, $\Sigma U_{1-2} = 0$, from equation (1).

Equation (2) $\Rightarrow \Sigma U_{1-2} = -\left(\frac{1}{2}k(0)^2 - \frac{1}{2}k(0.30)^2\right) - (15)(9.8)(0.90) = 0$

Solve for k:

$$k = 2940\ N/m$$

Resolve the above equation using this value of k, with $g_{moon} = 9.8/6\ m/s^2$, for Δd:

$$\Rightarrow \Sigma U_{1-2} = -\left(\frac{1}{2}(2940)(0)^2 - \frac{1}{2}(2940)(0.30)^2\right) - (15)\left(\frac{9.8}{6}\right)\Delta d = 0$$

Solve for Δd:

$$\Delta d = 5.4m$$

The maximum lift off height is $5.4m - s_1 = 5.4m - 0.30m = 5.1\ m$
(answer)