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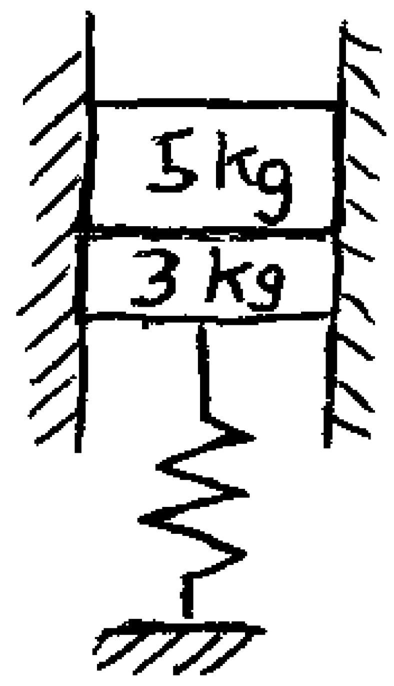
This is a problem involving work and energy.



A 5 kg mass is slowly placed on a 3 kg mass that is initially at rest while supported by a spring underneath. If the spring stiffness is 250 N/m, what is the maximum compression amount of the spring, and what is the maximum velocity of the masses?

Solution:

Draw free-body diagram for the 3 kg mass before the 5 kg mass is placed on top of it:

$\downarrow mg$

$\boxed{3\,kg}$ $\uparrow +$

$\uparrow F_s$ (spring force)

The mass is in static equilibrium so,

$$F_s - mg = 0$$
$$F_s = mg$$
$$F_s = (3)(9.8)$$
$$F_s = 29.4\ N$$

The initial deflection (compression) of the spring is

$$s_1 = \frac{F_s}{k} = \frac{29.4\,N}{250\,N/m} = 0.1176\,m$$

Apply the principle of work and energy:

$$T_1 + \sum U_{1-2} = T_2 \quad (1)$$

$$\Sigma U_{1-2} = \underbrace{-\left(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2\right)}_{\text{Work done by spring}} + \underbrace{W\Delta d}_{\text{Work done by gravity.}}$$

This is positive because gravity and displacement (Δd) of mass are in same direction

$W = mg$

$m = 3\,kg + 5\,kg = 8\,kg$

The final deflection of the spring is:

$s_2 = s_1 + \Delta d$

Substitute Known quantities into equation (1):

$$\frac{1}{2}mv_1^2 - \left(\frac{1}{2}k(s_1+\Delta d)^2 - \frac{1}{2}ks_1^2\right) + mg\Delta d = \frac{1}{2}mv_2^2$$

Substitute $m = 8\,kg$, $v_1 = 0$, $k = 250\,N/m$, and $s_1 = 0.1176\,m$:

$$-\left(125(0.1176+\Delta d)^2 - 125(0.1176)^2\right) + 78.4\,\Delta d = 4v_2^2$$

The maximum displacement of the mass occurs when $v_2 = 0$. Solve for Δd:

$$\Delta d = 0.392\,m$$

The maximum spring compression corresponds to the maximum displacement, which is $s_2 = 0.1176 + 0.392\,m$
$= 0.51\,m$
(answer)

The maximum velocity occurs at $\Delta d = 0.20\,m$, for which $v_2 = 1.095\,m/s$. (answer)

The algebra used to calculate this is not shown.