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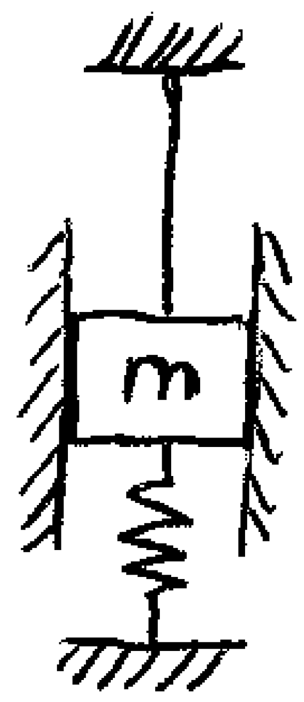
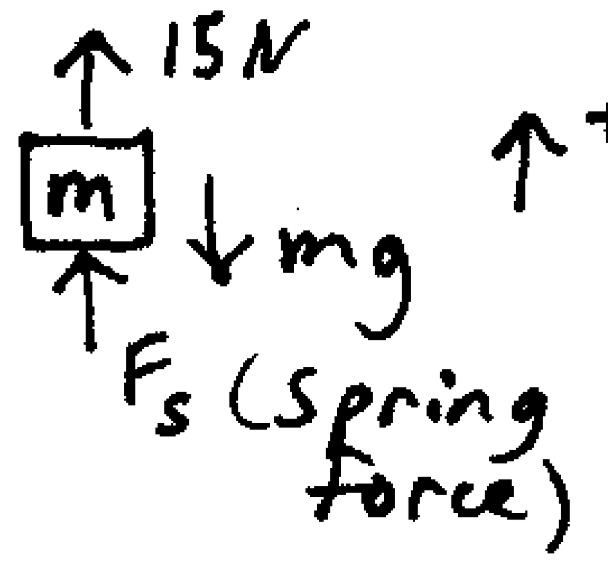
This is a problem involving work and energy.



A mass m is supported from above by a rope, and supported from below by a spring. The rope has a tension of 15 N, the spring stiffness is 900 N/m, and m = 4 Kg. If the rope is cut, determine the maximum displacement of the mass and determine the maximum velocity of the mass.

Solution:

Draw free-body diagram for mass when initially suspended:



Initially, the mass is in static equilibrium so,

$15 + F_s - mg = 0$

$F_s = -15 + mg$

$F_s = -15 + 4(9.8)$

$F_s = 24.2\ N$

The initial deflection (compression) of the spring is

$$s_1 = \frac{24.2\ N}{900\ N/m} = \frac{F_s}{K}$$

$s_1 = 0.0269\ m$

Apply the principle of work and energy:

$$T_1 + \Sigma U_{1-2} = T_2 \quad (1)$$

$$\Sigma U_{1-2} = \underbrace{-\left(\tfrac{1}{2}Ks_2^2 - \tfrac{1}{2}Ks_1^2\right)}_{\text{work done by spring}} + W\Delta d$$

$W\Delta d$: work done by gravity. This is positive because gravity and displacement (Δd) of mass are in same direction

$W = mg$

The final deflection of the spring is:

$$s_2 = s_1 + \Delta d$$

Substitute known quantities into equation (1):

$$\frac{1}{2}mv_1^2 - \left(\frac{1}{2}k(s_1+\Delta d)^2 - \frac{1}{2}ks_1^2\right) + mg\Delta d = \frac{1}{2}mv_2^2$$

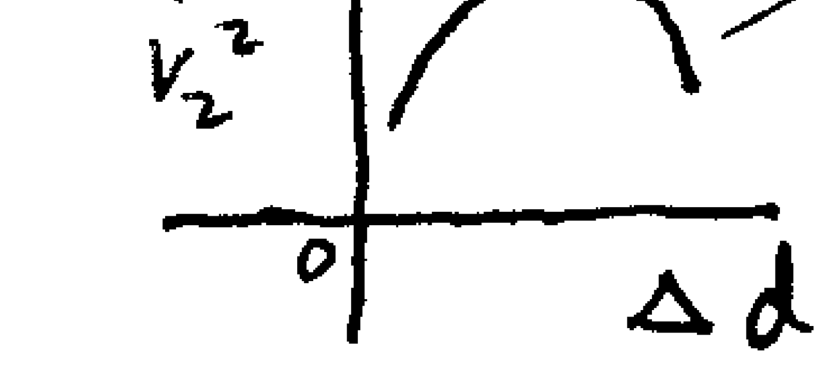
Substitute $m = 4\,kg$, $v_1 = 0$, $k = 900\,N/m$, and $s_1 = 0.0269\,m$:

$$\Rightarrow \boxed{-\left(450(0.0269+\Delta d)^2 - 450(0.0269)^2\right) + 39.2\Delta d = 2v_2^2} \quad (2)$$

The maximum displacement of the mass occurs when $v_2 = 0$. Solve for Δd:

$$\Delta d = 0.033\,m \quad \text{(answer)}$$

(ans.) The maximum velocity occurs at $\Delta d = 0.017\,m$ for which $v_2 = 0.25\,m/s$. The algebra for this is not shown, but basically you have to determine the maximum value of v_2^2 on a parabola that is in terms of Δd, in the above equation (2). Graphically, it looks like this:



parabolic graph of V_2^2 versus Δd.

The value of Δd for maximum v_2^2 is the same value of Δd for maximum v_2.