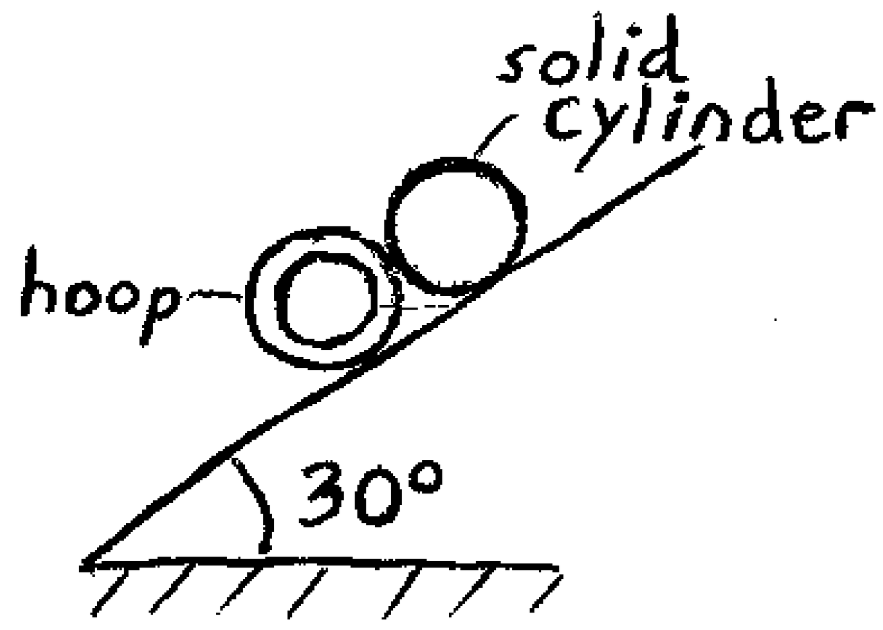
real-world-physics-problems.com
Oct. 15, 2020

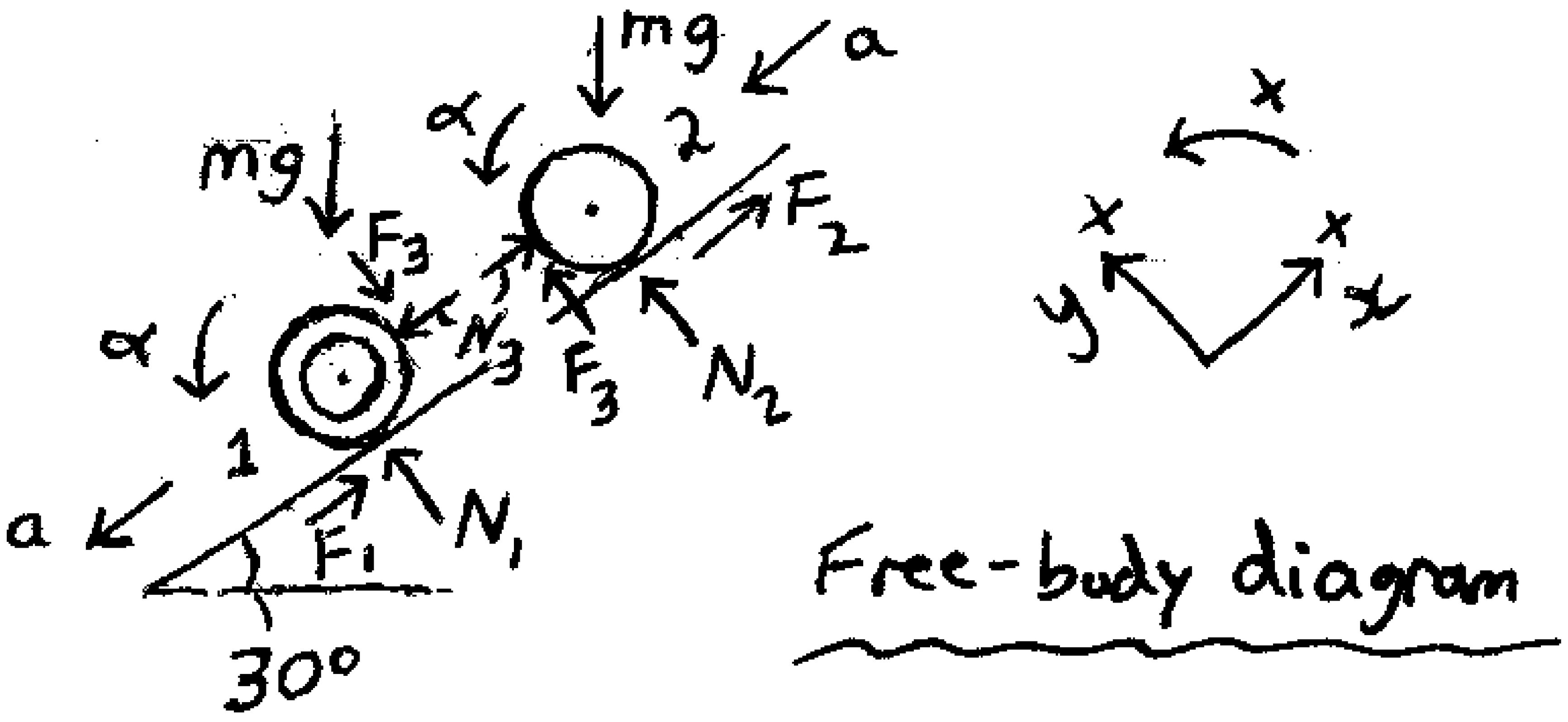
This is a force and motion problem involving rotation, rolling, and torque!



A hoop and solid cylinder are initially in contact on an incline, as shown. The mass of each is 50 Kg and the radius of each is 30 cm.

Is it possible for the hoop and cylinder to both remain stationary, and in contact, on the incline? If the coefficient of kinetic friction between the hoop and cylinder is 0.10, what is the angular acceleration of each, and what is the acceleration of the center of mass of each? How many rotations does the hoop and cylinder complete after each has rolled a distance of 1 meter?

Solution:



Free-body diagram

Assume hoop and cylinder do not slip on the incline.

Although the direction of a and α are shown, don't assign a direction for them in the dynamics analysis. Their direction will come out in the answer. Alternatively, you could assign a direction, but this way may be easier.

Case 1: Hoop and cylinder are stationary and are in contact. Then,

$\left(\begin{matrix} a=0 \\ \alpha=0 \end{matrix}\right)$ $\quad \left.\sum F_x = 0, \quad \sum F_y = 0, \quad \text{and} \quad \sum \tau_G = 0 \right\}$ static equilibrium requirement

G is the center of mass

For the hoop:

$\sum F_x = 0: \quad F_1 - N_3 - mg\sin 30° = 0 \quad (1)$, $m = 50\,\text{kg}$

$\sum F_y = 0: \quad N_1 - F_3 - mg\cos 30° = 0 \quad (2)$

$\sum \tau_G = 0: \quad F_1 R - F_3 R = 0$, $R = 0.30\,\text{m}$

$\Rightarrow F_1 = F_3 \quad (3)$

For the cylinder:

$\sum F_x = 0: \quad F_2 + N_3 - mg\sin 30° = 0 \quad (4)$, $m = 50\,\text{kg}$

$\sum F_y = 0: \quad F_3 + N_2 - mg\cos 30° = 0 \quad (5)$

$\sum \tau_G = 0: \quad F_2 R - F_3 R = 0$, $R = 0.30\,\text{m}$

$\Rightarrow F_2 = F_3 \quad (6)$

Solve equations (1)-(6):

$$\boxed{\begin{aligned} F_1 &= 245\,N \\ F_2 &= 245\,N \\ F_3 &= 245\,N \\ N_1 &= 669.35\,N \\ N_2 &= 179.35\,N \\ N_3 &= 0\,N \end{aligned}}$$

static friction needs to be present $\Big\{$ For the hoop and cylinder to both be stationary and in contact, there would have to be a sufficiently large normal force, N_3, between them such that $\frac{F_3}{N_3} \le \mu_s$.

But $N_3 = 0$, so

static friction not possible because $N_3 = 0$ $\Big\{$ this is not possible! (ans.)

Case 2: Hoop and cylinder are in contact and both rolling without slipping, on incline.

For the hoop:

$\Sigma F_x = m a_x$: $F_1 - N_3 - mg\sin 30° = ma$ (1)

$\Sigma F_y = m a_y$: $N_1 - F_3 - mg\cos 30° = 0$, since $a_y = 0$

Due to Kinetic Friction, $F_3 = \mu_K N_3$

$\Rightarrow N_1 - \mu_K N_3 - mg\cos 30° = 0$ (2) $\mu_K = 0.10$

$\Sigma \tau_G = I_G \alpha$: $F_1 R - F_3 R = I_G \alpha$, $I_G = mR^2$

$\Rightarrow F_1 - \mu_K N_3 = mR\alpha$

For rolling without slipping, $-a = \alpha R$

$\Rightarrow F_1 - \mu_K N_3 = -ma$ (3)

For the cylinder:

$\Sigma F_x = m a_x$: $F_2 + N_3 - mg\sin 30° = ma$ (4)

$\Sigma F_y = m a_y$: $F_3 + N_2 - mg\cos 30° = 0$, since $a_y = 0$

$\Rightarrow \mu_K N_3 + N_2 - mg\cos 30° = 0$ (5)

$\Sigma \tau_G = I_G \alpha$: $F_2 R - F_3 R = I_G \alpha$, $I_G = \frac{1}{2} mR^2$

$\Rightarrow F_2 - \mu_K N_3 = \frac{1}{2} mR\alpha$

For rolling without slipping, $-a = \alpha R$

$\Rightarrow F_2 - \mu_K N_3 = -\frac{1}{2} ma$ (6)

Both hoop and cylinder move with the same acceleration because they are in contact and have the same radius, R.

Solve equations (1) – (6):

$F_1 = 141.48\ N$
$F_2 = 72.46\ N$
$a = -2.76\ m/s^2$
$N_1 = 427.80\ N$
$N_2 = 420.90\ N$
$N_3 = 34.51\ N$

The acceleration of the center of mass of hoop and cylinder is $2.76\ m/s^2$ down the incline, and the angular acceleration of each is $\alpha = -\frac{(-2.76)}{R}$, so $\alpha = 9.2\ rad/s^2$ ↺ . (answer)

$R = 0.30\ m$

Next, find the time it takes the hoop and cylinder to roll a distance of 1 meter.

$$\Delta d = \underbrace{v_i}_{0} \Delta t + \frac{1}{2} a \Delta t^2 = 0 + \frac{1}{2}(2.76)\Delta t^2$$

(assume they start from rest)

For $\Delta d = 1\ m$,

$\Delta t = 0.851\ s$

Now, find the number of revolutions of the hoop and cylinder in $\Delta t = 0.851\ s$, starting from rest:

$$\Delta\theta = \underbrace{w_i}_{0} \Delta t + \frac{1}{2}\alpha\Delta t^2$$

$$\Delta\theta = 0 + \frac{1}{2}(9.2)(0.851)^2 = 3.33 \text{ radians},$$

which is $\frac{3.33}{2\pi} = 0.53$ revolutions (answer)