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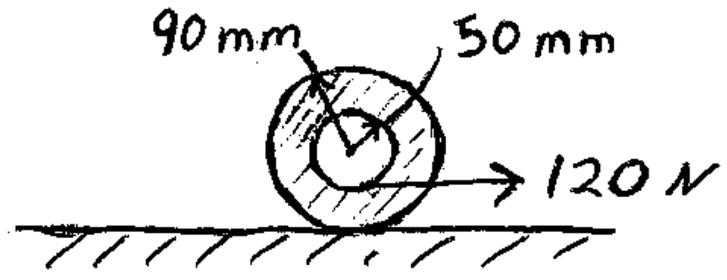
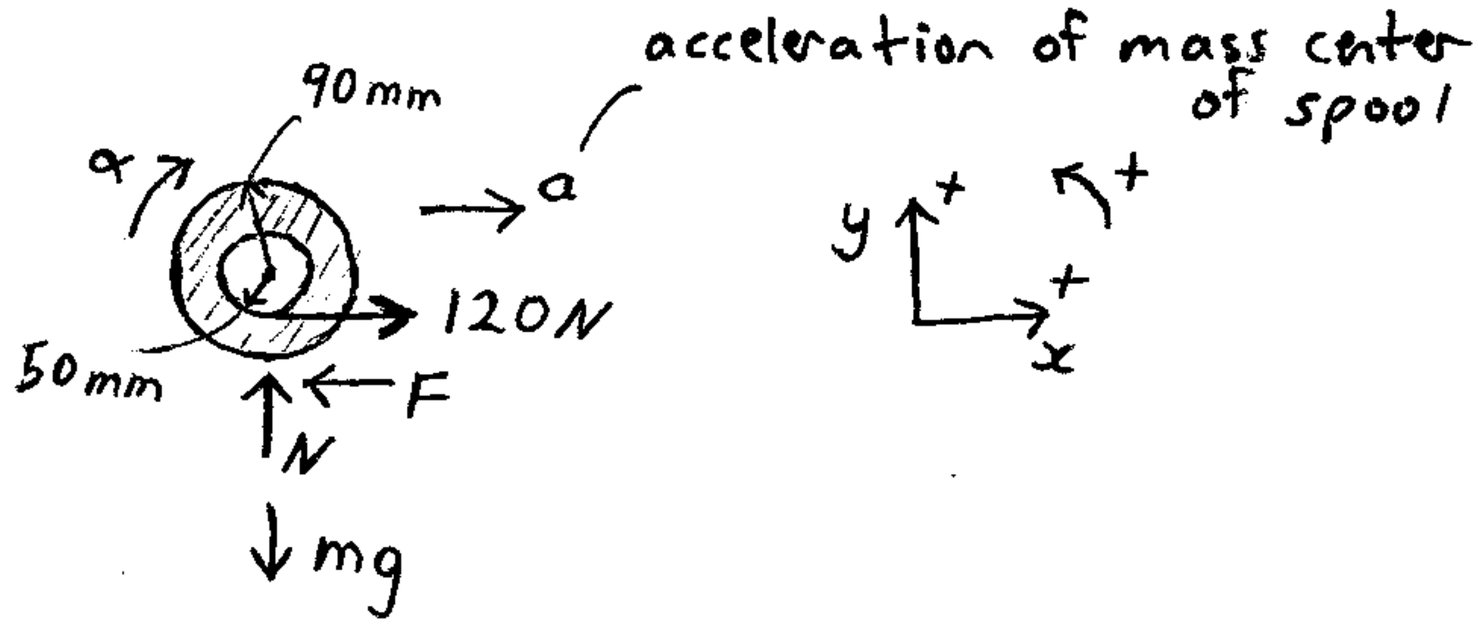
This is a force and motion problem involving rotation, rolling, and torque.



A cord is wound around the inner drum of a spool and pulled with a horizontal force of 120 N. The mass of the spool is 40 kg, and its radius of gyration is 60 mm. If $\mu_s = 0.15$ and $\mu_k = 0.10$, determine the angular acceleration of the spool and the acceleration of its center of mass.

Solution:



Although the direction of a and α are shown, don't assign a direction for a and α in equations (1) and (3). Their direction will come out in the answer.

Apply Newton's second law:

$$\Sigma F_x = m a_x \quad , m = 40\,kg$$

$$\Rightarrow \quad 120 - F = ma \quad (1)$$

$$\Sigma F_y = m a_y \quad , \quad a_y = 0$$

$$\Rightarrow \quad N - mg = 0$$

$$\Rightarrow N = mg \quad (2)$$

Apply Newton's second law in angular form:

$$\Sigma \tau_G = I_G \alpha$$

$$\Rightarrow \quad 120\,(0.05) - F\,(0.09) = I_G \alpha \quad (3)$$

$$I_G = m k^2$$

k is the radius of gyration

$$k = 0.06\,m$$

$$I_G = 40\,(0.06)^2$$

$$I_G = 0.144 \; kg \cdot m^2$$

For no slipping, $a = -\alpha R$

$$R = 0.09\,m$$

(based on the sign convention)

$$\Rightarrow a = -0.09\alpha \quad (4)$$

Solve equations (1), (3), (4):

$$a = 0.923 \; m/s^2$$
$$\alpha = -10.256 \; rad/s^2$$
$$F = 83.077 \; N$$

Check: $\frac{F}{N} = \frac{83.077}{40(9.8)} = 0.21 > \mu_s \,, \; \mu_s = 0.15$

Therefore, the spool slips on the ground and the above solution is not valid.

The problem must be resolved given that the spool slips on the ground.

Kinetic friction

In equation (1), substitute $F = \mu_K N$, $N = mg$ from equation (2)

$\Rightarrow F = 0.10(40)(9.8) = 39.2\ N$

$(1) \Rightarrow 120 - 39.2 = 40a$

Solve: $a = 2.02\ m/s^2$ (answer)

Similarly, in equation (3), substitute $F = 39.2\ N$ and solve for α:

$\Rightarrow 120(0.05) - 39.2(0.09) = 0.144\alpha$

Solve: $\alpha = 17.167\ rad/s^2$ (answer)

Reminder: When there is Kinetic friction, due to sliding, you have to know the direction of the Friction. In this problem it is acting to the left. But when there is static friction, such as for no slipping, you don't have to know its direction - it Comes out in the answer.