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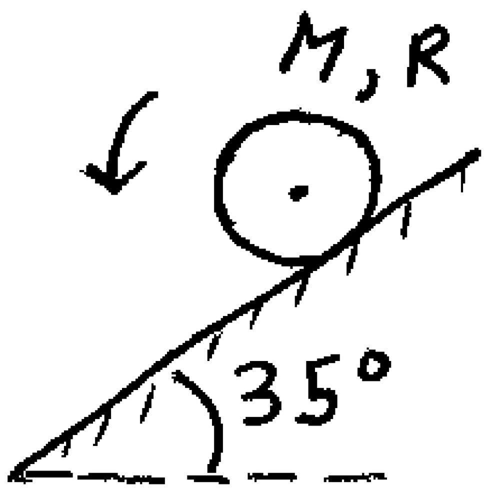
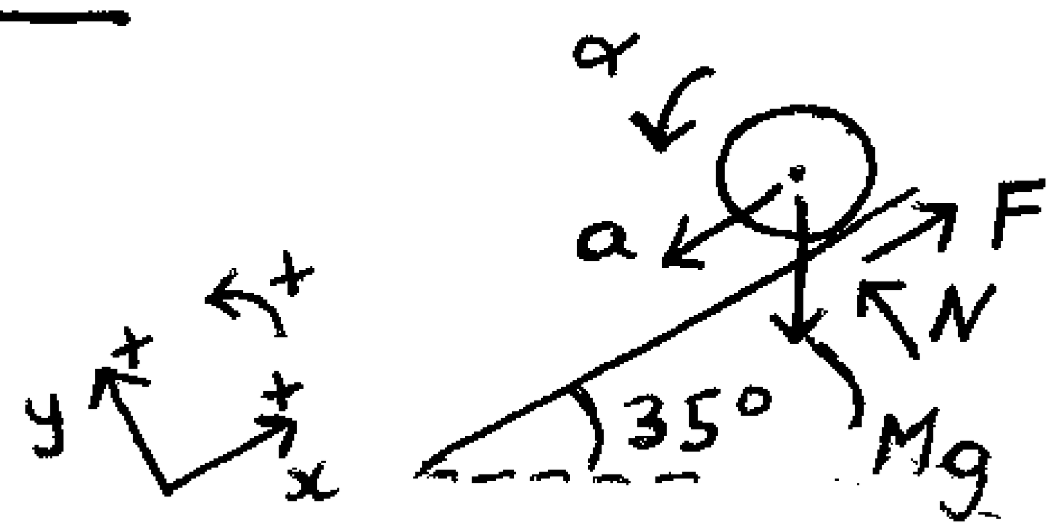
This is a force and motion problem involving rotation, rolling, and torque.



A cylinder is rolling down an incline as shown. What is the minimum coefficient of static friction between the cylinder and surface so that the cylinder rolls without slipping? Suppose the coefficient of kinetic friction between the cylinder and surface is 0.10. What is the angular acceleration of the cylinder and what is the acceleration of its center of mass? Compare this to the no-slipping case.

Note that $M = 10\text{ kg}$ and $R = 6\text{ cm}$.

Solution:



Apply Newton's second law:

$$\Sigma F_x = m a_x$$

$$\Rightarrow -Mg\sin 35° + F = Ma \quad (1)$$

Apply Newton's second law in angular form:

$$\Sigma \tau_G = I_G \alpha \Rightarrow FR = I_G \alpha \quad (2)$$

Although the direction of a and α are shown, don't assign a direction for a and α in equations (1) and (2). Their direction will come out in the answer.

From the Kinematics,

$\Rightarrow a = -\alpha R$ (3) , for no slipping case

and

$\Rightarrow F = \mu_K N$ (4), for the case where the cylinder slips on the incline

Apply Newton's second law in y-direction:

$\Sigma F_y = m a_y$, $a_y = 0$

$\Rightarrow N - Mg\cos 35° = 0$

$\Rightarrow N = Mg\cos 35°$ (5)

For the no slipping case, combine equations (1), (2), (3), (5), and solve for F and N, and $\frac{F}{N}$.

$\Rightarrow -Mg\sin 35° + F = M(-\alpha R)$

$\Rightarrow -Mg\sin 35° + F = -MR\left(\frac{FR}{I_G}\right)$

$$\Rightarrow F = \frac{Mg\sin 35°}{1 + \frac{MR^2}{I_G}}$$

$\Rightarrow N = Mg\cos 35°$

$$\Rightarrow \frac{F}{N} = \frac{\dfrac{Mg\sin 35^\circ}{1 + \dfrac{MR^2}{I_G}}}{Mg\cos 35^\circ}$$

substitute $I_G = \frac{1}{2}MR^2$.

$$\Rightarrow \frac{F}{N} = \frac{\dfrac{Mg\sin 35^\circ}{1+2}}{Mg\cos 35^\circ}$$

$$\Rightarrow \frac{F}{N} = \frac{\tan 35^\circ}{3} = 0.233$$, this is the minimum coefficient of static friction (answer)

For the slipping case, combine equations (1), (2), (4), (5), and solve for a and α.

$$\Rightarrow -Mg\sin 35^\circ + \mu_K N = Ma$$

$$\Rightarrow -Mg\sin 35^\circ + \mu_K Mg\cos 35^\circ = Ma$$

$$\Rightarrow a = \mu_K g\cos 35^\circ - g\sin 35^\circ$$

$$\Rightarrow a = (0.10)(9.8)\cos 35^\circ - (9.8)\sin 35^\circ$$

$a = -4.82\ m/s^2$ (answer)

$$\Rightarrow \mu_K N R = I_G \alpha$$

$$\Rightarrow \alpha = \frac{\mu_K N R}{I_G} = \frac{\mu_K Mg\cos 35^\circ R}{\frac{1}{2}MR^2}$$

$$\Rightarrow \alpha = \frac{(0.10)(9.8)\cos 35°}{\frac{1}{2}(0.06)} = 26.76 \text{ rad/s}^2 \text{ (answer)}$$

For the no slipping case,

$$\Rightarrow Ma = \frac{Mg\sin 35°}{3} - Mg\sin 35°$$

$$\Rightarrow a = -\frac{2}{3} g\sin 35° = -3.75 \text{ m/s}^2$$

(less than for the slipping case)

$$\Rightarrow \alpha = \frac{\frac{Mg\sin 35°}{3} \cdot R}{I_G}$$

$$\Rightarrow \alpha = \frac{Mg\sin 35° \cdot R}{3 \cdot \frac{1}{2} MR^2}$$

$$\Rightarrow \alpha = \frac{2}{3}\frac{g\sin 35°}{R}$$

$$\Rightarrow \alpha = \frac{2}{3}\frac{(9.8)\sin 35°}{0.06} = 62.46 \text{ rad/s}^2$$

(greater than for slipping case)