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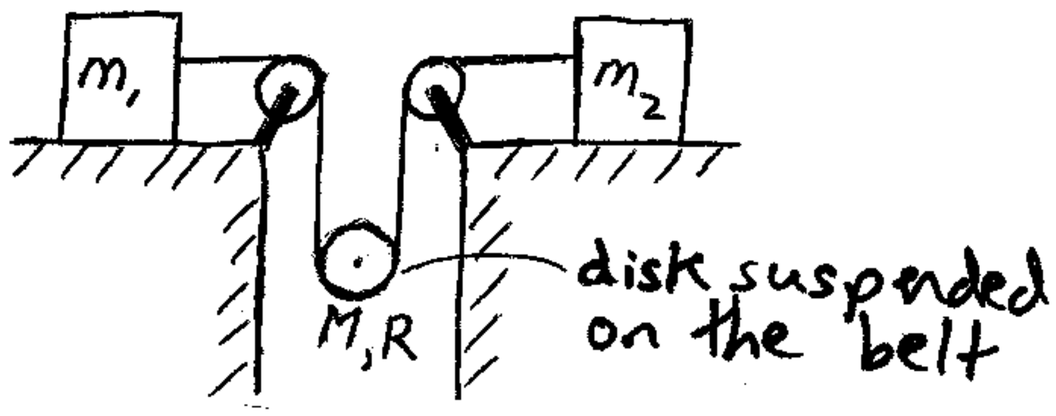
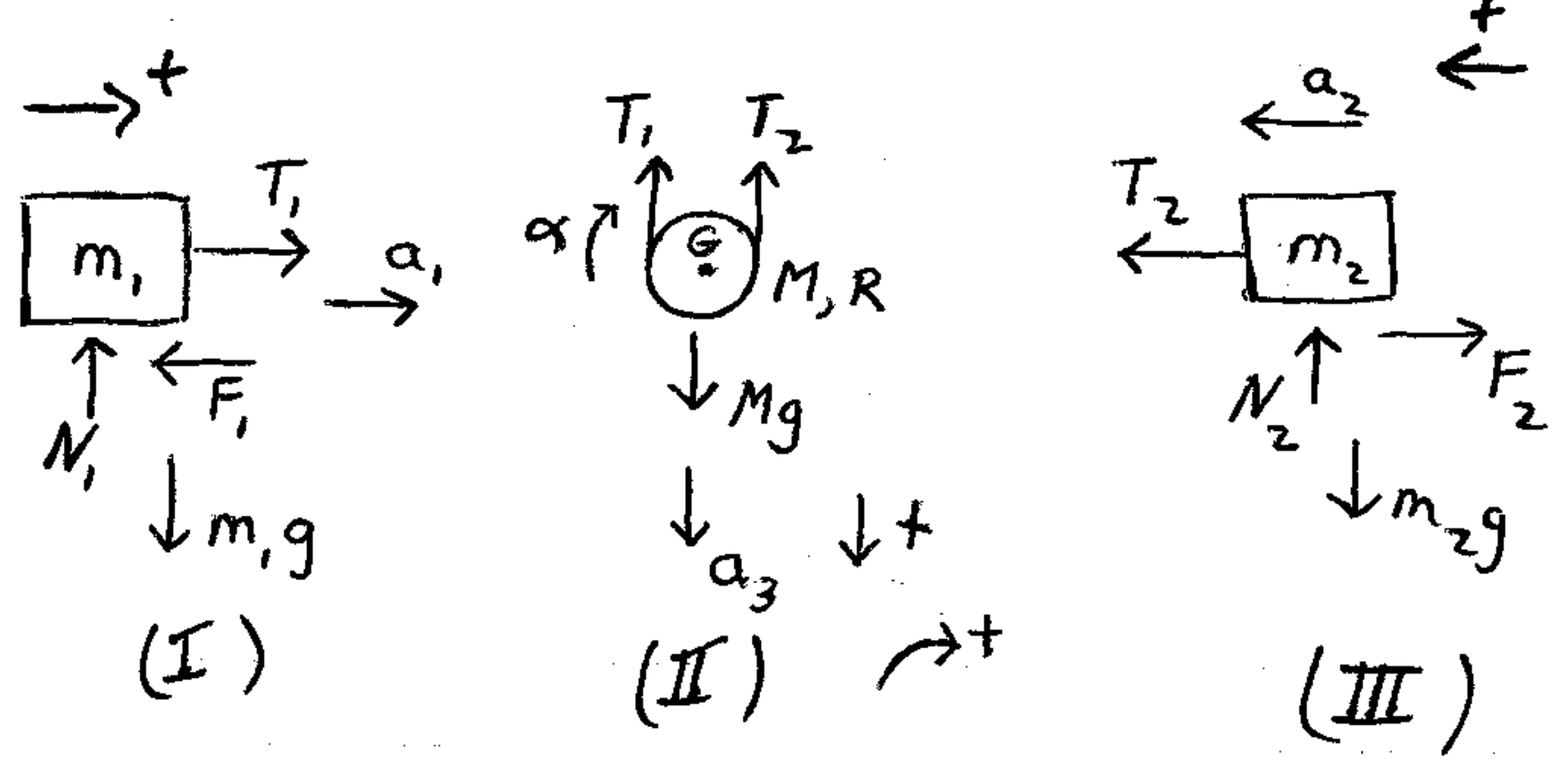
This is a force and motion problem involving rotation, rolling, and torque.



In the pulley system shown, $m_1 = 2\,kg$, $m_2 = 3\,kg$, $M = 5\,kg$, $R = 5\,cm$, and the coefficient of kinetic friction between the blocks and surface is 0.15. The mass of belt and pulleys is negligible. Determine the angular acceleration of the disk and the acceleration of its center of mass.

Solution:



From the kinematics,

(1) $a_1 + a_2 = 2a_3$, $a_3 = a_G$ (acceleration of center of mass of disk)

assumes belt does not slip on disk:

$a_1 = a_3 - \alpha R$

$a_2 = a_3 + \alpha R$

$\Rightarrow a_2 - a_1 = 2\alpha R$ (2)

(I) Apply Newton's second law:

$$\Sigma F_x = m a_x$$

$$\Rightarrow T_1 - F_1 = m_1 a_1 \quad , \quad F_1 = \mu_K N_1$$

$$\Sigma F_y = m a_y \quad , \quad a_y = 0$$

$$\Rightarrow N_1 - m_1 g = 0 \Rightarrow N_1 = m_1 g$$

$$\Rightarrow T_1 - \mu_K m_1 g = m_1 a_1 \quad (3)$$

(II) Apply Newton's second law in angular form:

$$\Sigma \tau_G = I_G \alpha \quad , \quad I_G = \frac{1}{2} M R^2 , \; M = 5kg$$

$$\Rightarrow T_1 R - T_2 R = I_G \alpha \quad (4)$$

Note: R is the radius of the disk, $R = 0.05m$

Apply Newton's second law: $\Sigma F_y = m a_y$

$$-T_1 - T_2 + Mg = M a_3 \quad (5)$$

(III) Apply Newton's second law:

$$\Sigma F_x = m a_x$$

$$\Rightarrow T_2 - F_2 = m_2 a_2 \quad , \quad F_2 = \mu_K N_2$$

$$\Sigma F_y = m a_y \quad , \quad a_y = 0$$

$$\Rightarrow N_2 - m_2 g = 0 \Rightarrow N_2 = m_2 g$$

$$\Rightarrow T_2 - \mu_K m_2 g = m_2 a_2 \quad (6)$$

Substitute $m_1 = 2kg$, $m_2 = 3kg$, $M = 5kg$, $R = 0.05m$, $\mu_K = 0.15$, and $I_G = \frac{1}{2}(5)(0.05)^2$ into equations (1)–(6) and solve the unknowns:

(ans.) $a_1 = 5.0 m/s^2$, $a_2 = 3.48 m/s^2$, $a_3 = 4.24 m/s^2$, $\alpha = -15.23 rad/s^2$, $T_1 = 12.95N$, $T_2 = 14.85N$