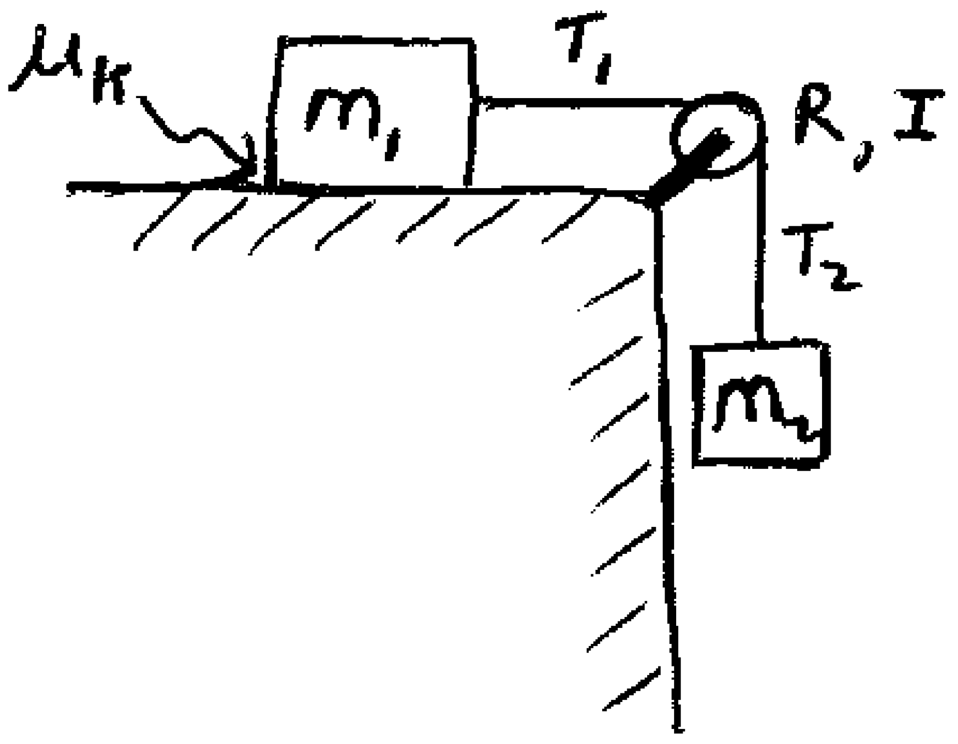
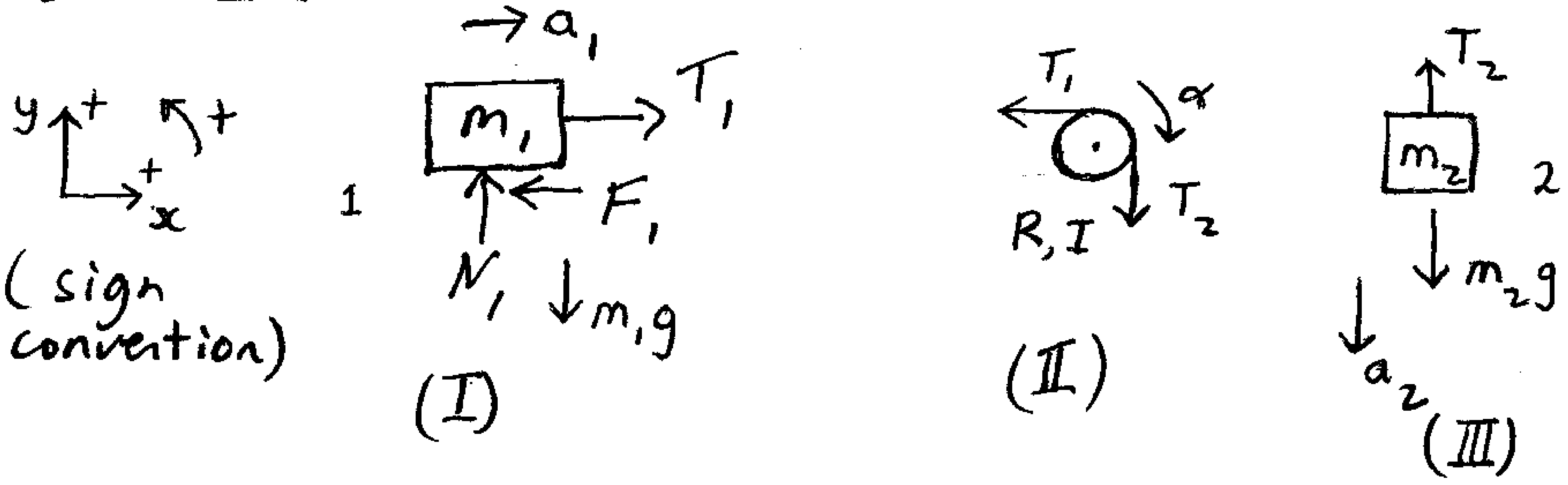
This is a force and motion problem involving rotation, rolling, and torque.



In the pulley system shown, the two blocks have a mass m_1 and m_2, the pulley radius is R, the rotational inertia of the pulley is I, and the coefficient of kinetic friction between the block and surface is μ_K. Determine (a) the angular acceleration of the pulley, (b) the acceleration of the blocks, and (c) the tensions in the upper and lower sections of the rope.

Assume the system is in motion.

Solution:



Note that I is the rotational inertia of the pulley about its center of mass.

(I) Apply Newton's second law:

$$\Sigma F_x = m a_x$$

$$\Rightarrow T_1 - F_1 = m_1 a_1 \quad , \quad F_1 = \mu_K N_1$$

$$\Sigma F_y = m a_y \ , \quad a_y = 0$$

$$\Rightarrow N_1 - m_1 g = 0 \Rightarrow N_1 = m_1 g$$

So, $F_1 = \mu_K m_1 g$ and,

$$\Rightarrow T_1 - \mu_K m_1 g = m_1 a_1 \quad (1)$$

(II) Apply Newton's second law in angular form:

$$\Sigma \tau = I\alpha \Rightarrow \underbrace{T_1 R - T_2 R}_{\text{torque about center of mass of pulley}} = I(-\alpha)$$

$$\Rightarrow T_1 R - T_2 R = -I\alpha \quad (2)$$

(III) Apply Newton's second law:

$$\Sigma F_y = m a_y$$

$$\Rightarrow T_2 - m_2 g = m_2(-a_2)$$

$$\Rightarrow T_2 - m_2 g = -m_2 a_2 \quad (3)$$

Assume the rope does not slip over the pulley, so that,

$$a_1 = a_2 = a = R\alpha \quad (4)$$

From equation (1),

$$T_1 = m_1 a + \mu_K m_1 g$$

sub. eq. (4) $\Rightarrow T_1 = m_1(R\alpha) + \mu_K m_1 g \quad (5)$

From equation (3),

$$T_2 = -m_2 a + m_2 g$$

sub eq. (4) $\Rightarrow T_2 = -m_2(R\alpha) + m_2 g \quad (6)$

Substitute the above two equations for T_1 and T_2 into equation (2):

$$(m_1 R\alpha + \mu_K m_1 g)R - (-m_2 R\alpha + m_2 g)R = -I\alpha$$

Solve: $\alpha = \dfrac{R(m_2 g - \mu_K m_1 g)}{m_1 R^2 + m_2 R^2 + I}$ (answer for (a))

The acceleration of the blocks is $a = R\alpha$,

so $a = \dfrac{R^2(m_2 g - \mu_K m_1 g)}{m_1 R^2 + m_2 R^2 + I}$ (answer for (b))

From equation (5),

$$T_1 = \frac{m_1 R^2 (m_2 g - \mu_K m_1 g)}{m_1 R^2 + m_2 R^2 + I} + \mu_K m_1 g$$

From equation (6),

$$T_2 = \frac{- m_2 R^2 (m_2 g - \mu_K m_1 g)}{m_1 R^2 + m_2 R^2 + I} + m_2 g$$

answer for (c)

Note that, rather than assign a direction for acceleration of the two blocks, and a direction for angular acceleration of the pulley, you can just do the following:

$$\left.\begin{aligned} a_1 &= -R\alpha \\ a_2 &= R\alpha \end{aligned}\right\}$$ based on the kinematics of the problem and the sign convention.

So, $\Sigma F_x = m_1 a_1$ for block 1 $\Rightarrow T_1 - \mu_K m_1 g = -m_1 R \alpha$

$\Sigma \tau = I\alpha$ for pulley $\Rightarrow T_1 R - T_2 R = I\alpha$

$\Sigma F_y = m_2 a_2$ for block 2 $\Rightarrow T_2 - m_2 g = m_2 R \alpha$

You get the same answer as before, and the direction for the acceleration of the two blocks, and the direction for the angular acceleration of the pulley comes out of the answer.