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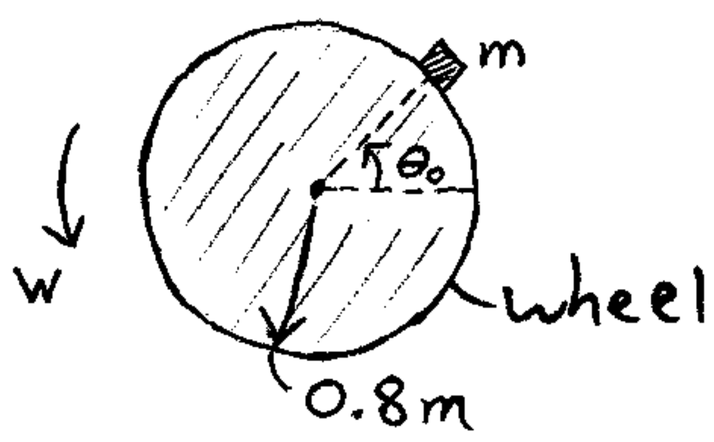
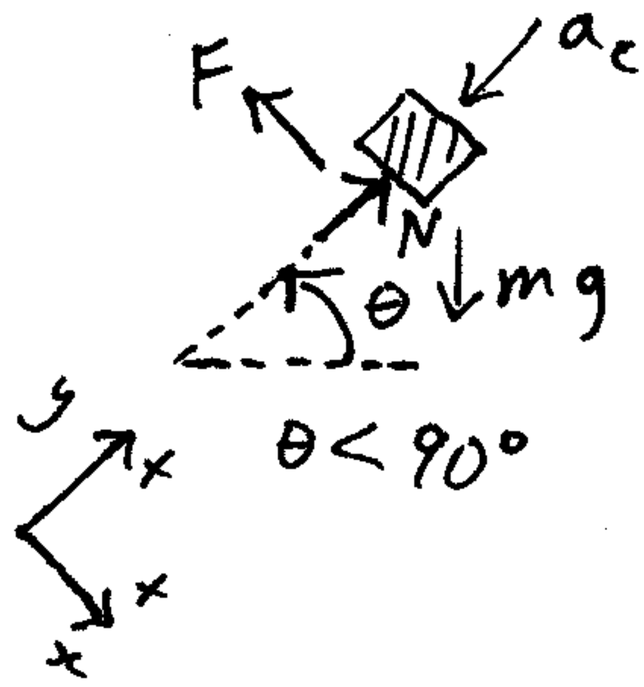
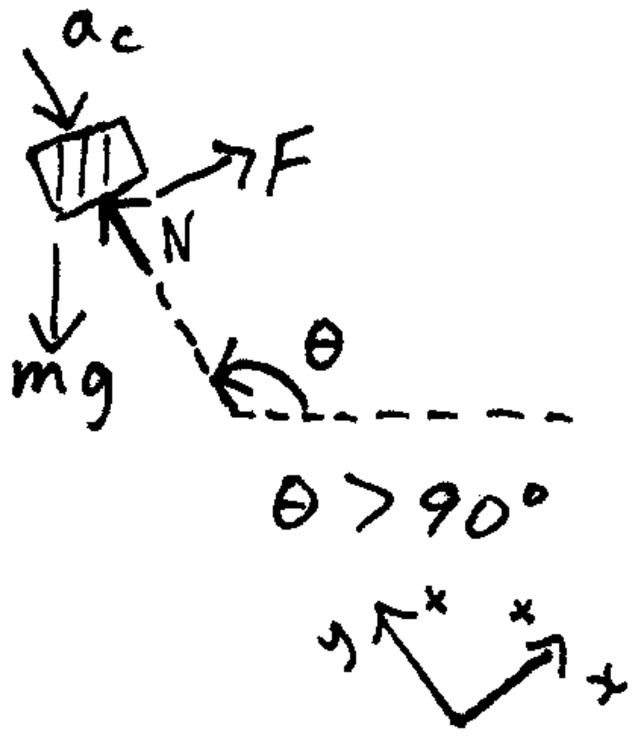
This is a force and motion problem involving uniform circular motion.



A block of mass m is placed on a wheel oriented vertically, rotating with an angular velocity of $w = 2\ rad/s$. If the block is placed initially with no slippage on the wheel, at what angle θ_0 can the block be placed so that the block stays on the wheel, without losing contact or slipping, for as long as possible? The coefficient of static friction is 0.30.

Solution:





Case 1, $\theta < 90°$:

Apply Newton's second law in the x-direction:

$$-F + mg\cos\theta = ma_x \quad , \; a_x = 0$$

$$\Rightarrow F = mg\cos\theta \quad (1)$$

Apply Newton's second law in the y-direction:

$$N - mg\sin\theta = ma_y \quad , \; a_y = -a_c$$

$$\Rightarrow N = mg\sin\theta - mw^2R \quad (2) \qquad a_c = w^2R$$

For no slipping, $\frac{F}{N} \leq \mu_s$

For the limiting case, determine the angle θ for which,

$$\frac{F}{N} = \mu_s$$

substitute equations (1) and (2) in the above equation:

$$\frac{g\cos\theta}{g\sin\theta - w^2R} = \mu_s \quad , \; \mu_s = 0.30$$

substitute given values:

$$\frac{9.8\cos\theta}{9.8\sin\theta - (2)^2(0.8)} = 0.30$$

Solve, $\theta = 79°$. For $\theta < 79°$ the block slips and falls off.

Case 2, $\theta > 90°$:

Apply Newton's second law in the x-direction:

$$F + mg\cos\theta = ma_x \quad , \quad a_x = 0$$

$$\Rightarrow F = -mg\cos\theta \quad (3)$$

Apply Newton's second law in the y-direction:

$$N - mg\sin\theta = ma_y \quad , \quad a_y = -a_c \quad a_c = w^2R$$

$$\Rightarrow N = mg\sin\theta - mw^2R \quad (4)$$

For no slipping, $\frac{F}{N} \leq \mu_s$

For the limiting case, determine the angle θ for which,

$$\frac{F}{N} = \mu_s$$

Substitute equations (3) and (4) in the above equation:

$$\frac{-g\cos\theta}{g\sin\theta - w^2R} = \mu_s \; , \; \mu_s = 0.30$$

substitute given values and solve, $\theta = 101°$.

∴ The block starts sliding at $\theta = 101°$, and the block stays on the wheel, without slipping, for $79° \leq \theta \leq 101°$. So the block stays on as long as possible, for $\theta_0 = 79°$. (answer)