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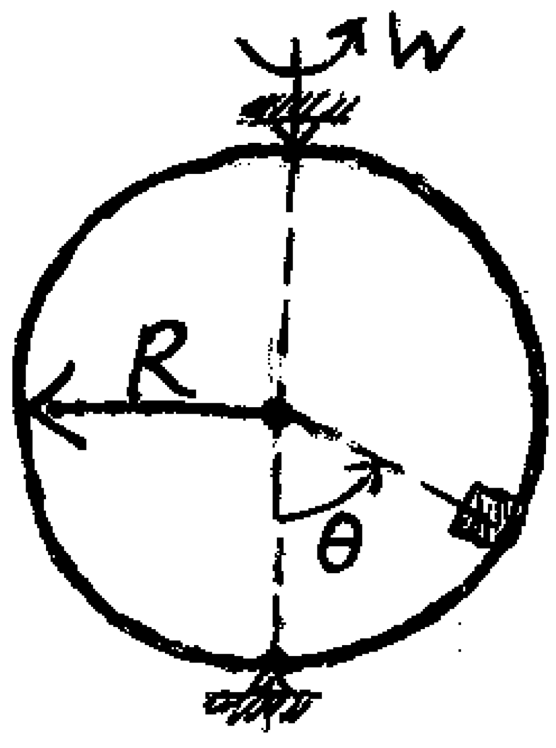
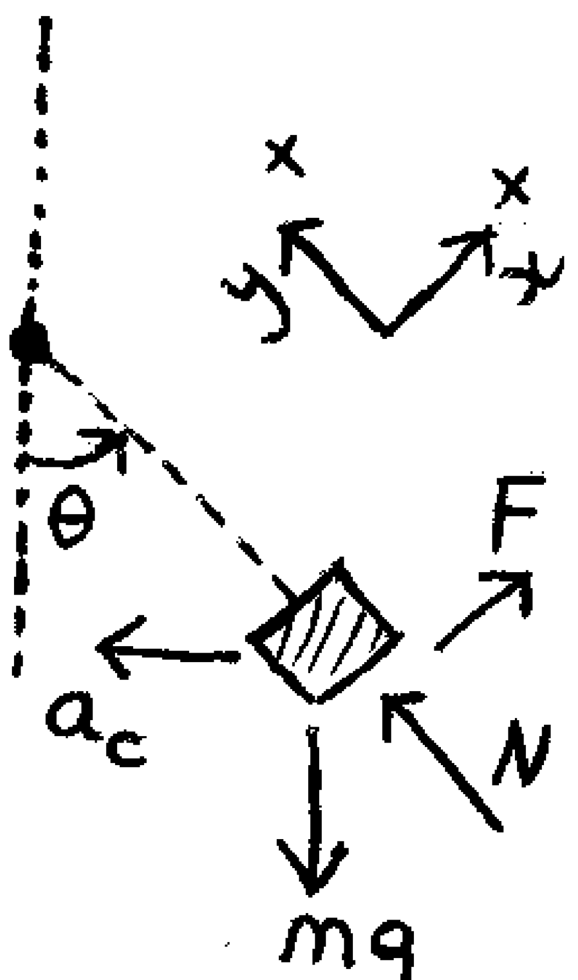
This is a force and motion problem involving uniform circular motion.



A hollow sphere with a block of mass 0.500 Kg inside of it is rotating with an angular velocity, ω, of 8.5 rad/s. If the inside radius of the sphere is R = 0.70 m, determine the values for θ if (a) there is no friction between the block and sphere and the block stays in contact with the sphere wall and does not slide on the sphere wall, (b) the coefficient of static friction between the block and sphere is 0.25 and the block stays in contact with the sphere wall and does not slide on the sphere wall.

Solution:



Free-body diagram

Apply Newton's second law in x-direction:

$$F - mg\sin\theta = m(-a_c\cos\theta) \quad , a_c = w^2 R_c$$
$$R_c = R\sin\theta$$

$$\Rightarrow F - mg\sin\theta = -mw^2 R\sin\theta\cos\theta \quad (1)$$

Apply Newton's second law in y-direction:

$$N - mg\cos\theta = m(a_c\sin\theta)$$

$$\Rightarrow N - mg\cos\theta = mw^2 R\sin^2\theta \quad (2)$$

(a) For no friction, $F=0$, and equation (1) reduces to:

$$g = w^2 R\cos\theta$$

substitute known values:

$$\Rightarrow 9.8 = (8.5)^2(0.70)\cos\theta$$

$$\theta = 78.8° \quad \text{(answer)}$$

The other value of θ that would result in the block staying in contact and not sliding is $\theta = 0°$, which satisfies equation (1) for $F=0$.

(b) First, determine the angles θ for which the block stays in contact and doesn't slide for the limiting static friction case, for $F = \mu_s N$, in the direction shown, and for $F = \mu_s N$, in opposite the direction shown.

For $F = \mu_s N$ in the direction shown,

equation (1) $\Rightarrow \mu_s N - mg\sin\theta = -m\omega^2 R \sin\theta\cos\theta$

combine this with equation (2) to eliminate N:

$$g(\sin\theta - \mu_s\cos\theta) = \omega^2 R(\mu_s \sin^2\theta + \sin\theta\cos\theta)$$

substitute given values:

$$9.8(\sin\theta - 0.25\cos\theta) = (8.5)^2(0.70)(0.25\sin^2\theta + \sin\theta\cos\theta)$$

Solve, $\theta = 93.0°$

Next, for $F = \mu_s N$ in opposite the direction shown,

equation (1) $\Rightarrow -\mu_s N - mg\sin\theta = -m\omega^2 R\sin\theta\cos\theta$

combine this with equation (2) to eliminate N:

$$g(\sin\theta + \mu_s\cos\theta) = \omega^2 R(-\mu_s\sin^2\theta + \sin\theta\cos\theta)$$

Substitute given values and solve, $\theta = 3.5°, 63.8°, 177.7°$

We're not there yet. For this tricky problem it's now helpful to determine the values of F and N, from equations (1) and (2) on page 2, from $0°$ to $180°$ in $20°$ increments, along with the corresponding ratio $|F|/N$:

Values for θ:

$0°$: $F=0$, $N=4.9\,N$, $|F|/N = 0$
$20°$: $F=-6.45\,N$, $N=7.56\,N$, $|F|/N = 0.85$
$40°$: $F=-9.30\,N$, $N=14.2\,N$, $|F|/N = 0.65$
$60°$: $F=-6.71\,N$, $N=21.42\,N$, $|F|/N = 0.31$
$80°$: $F=0.50\,N$, $N=25.38\,N$, $|F|/N = 0.02$
$100°$: $F=9.15\,N$, $23.67\,N$, $|F|/N = 0.39$
$120°$: $F=15.19\,N$, $N=16.5\,N$, $|F|/N = 0.92$
$140°$: $F=15.60\,N$, $N=6.69\,N$, $|F|/N = 2.3$
$160°$: $F=9.80\,N$, $N=-1.64\,N$ (no good, N must be >0)
$180°$: $F=0$, $N=-4.9\,N$ (no good, N must be >0)

(I) Between $0°$ and $20°$, $\frac{|F|}{N}$ increases from 0 to 0.85.
$\theta = 3.5°$ is a solution (from page 3) in this interval, which means that $0° \leq \theta \leq 3.5°$ is an interval for θ for which the block stays in contact and does not slide, since in this interval the static friction force never exceeds the maximum allowable, which is $\mu_s N$.

(II) Between $60°$ and $80°$, $\frac{|F|}{N}$ decreases from 0.31 to 0.02.
$\theta = 63.8°$ is a solution (from page 3), within this interval.

Between $80°$ and $100°$, $\frac{|F|}{N}$ increases from 0.02 to 0.39.
$\theta = 93.0°$ is a solution (from page 3), within this interval.

This means that $63.8° \leq \theta \leq 93.0°$ is an interval for θ for which the block stays in contact and does not slide, since in this interval the static friction force never exceeds the maximum allowable, which is $\mu_s N$ ($\mu_s = 0.25$)

For values of $\theta > 93.0°$, $|F|/N$ keeps increasing beyond 0.25 which means the block will slide, since the static friction force required to hold it in place exceeds the maximum allowable, which is $\mu_s N$ ($\mu_s = 0.25$).

Therefore, the values of θ for which the block does not slide and stays in contact with sphere wall are in the following intervals:

$$0° \leq \theta \leq 3.5°$$

and (answer for (b))

$$63.8° \leq \theta \leq 93.0°$$