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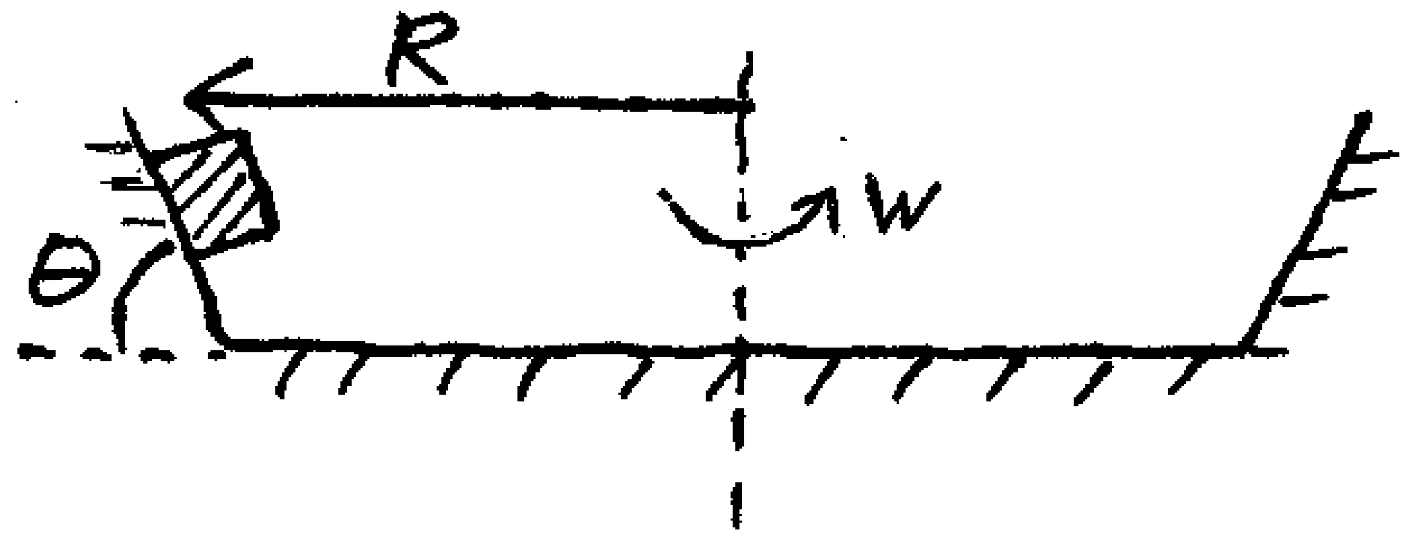
Oct. 7, 2020

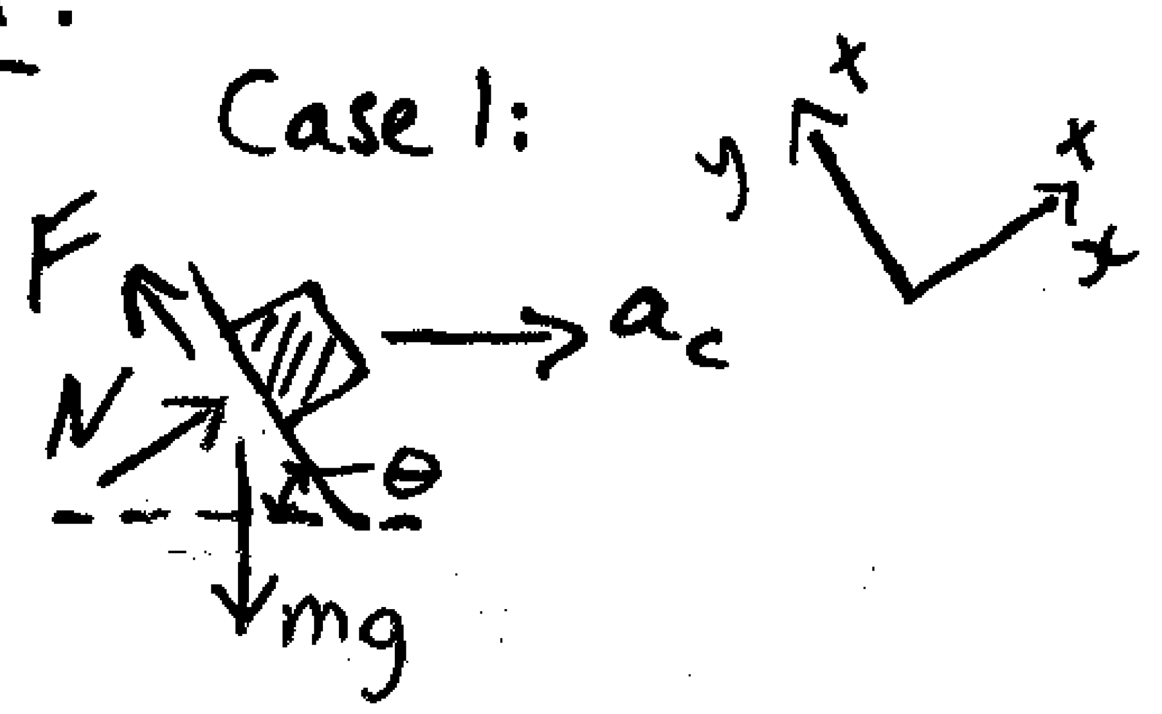
This is a force and motion problem involving uniform circular motion.



A person riding a Gravitron at an amusement park can be modelled as a block, as shown. It is given that $\theta = 70°$, $R = 6$ m, and the coefficient of static friction between the person and the wall of the Gravitron is 0.25. What is the range of angular velocities, w, of the Gravitron so that the person does not slide up or down the wall?

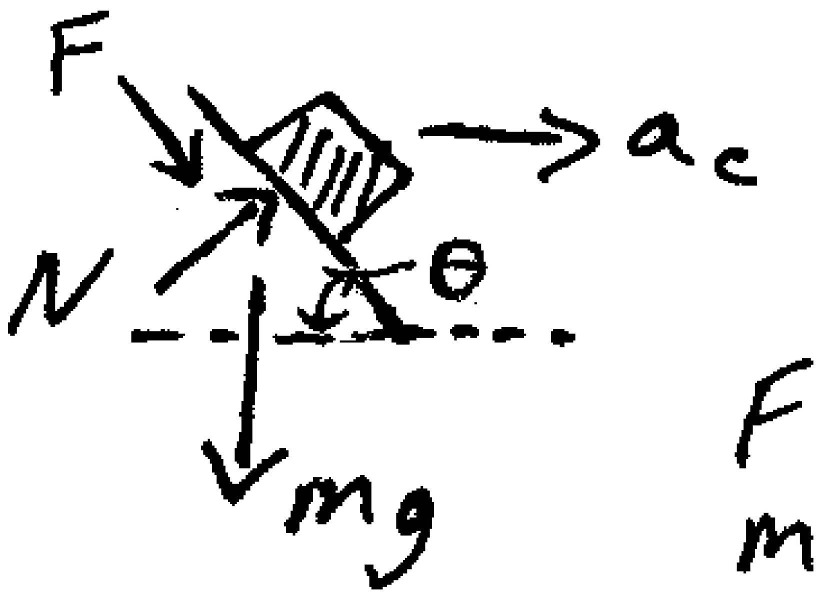
Solution:

Case 1:



Find minimum w so that person doesn't slide down the wall

Case 2:



Find maximum w so that person doesn't slide up the wall

Case 1: Apply Newton's second law in x-direction:

$$N - mg\cos\theta = m a_c \sin\theta \ , \ a_c = w^2 R$$

$$\Rightarrow N - mg\cos\theta = m w^2 R \sin\theta \quad (1)$$

Apply Newton's second law in y-direction:

$$F - mg\sin\theta = m(-a_c\cos\theta)$$

$$\Rightarrow F - mg\sin\theta = -m w^2 R\cos\theta \quad (2)$$

Find w when the static friction force is maximized, when $F = \mu_s N$. This is the minimum value of w so that the person does not slide down the wall. Substitute $F = \mu_s N$ into equation (2):

$$\Rightarrow \mu_s N - mg\sin\theta = -m w^2 R\cos\theta$$

Combine this with equation (1) to solve for w:

$$w = \sqrt{\frac{g(\sin\theta - \mu_s\cos\theta)}{R(\mu_s\sin\theta + \cos\theta)}}$$

For $g = 9.8\ m/s^2$, $R = 6\ m$, $\mu_s = 0.25$, $\theta = 70°$

$w = 1.56\ rad/s$ (minimum) (answer)

Case 3: Apply Newton's second law in x-direction:

$$N - mg\cos\theta = m a_c \sin\theta, \quad a_c = w^2 R$$

$$\Rightarrow N - mg\cos\theta = m w^2 R \sin\theta \quad (1)$$

Apply Newton's second law in y-direction:

$$-F - mg\sin\theta = m(-a_c\cos\theta)$$

$$\Rightarrow -F - mg\sin\theta = -m w^2 R\cos\theta \quad (2)$$

Find w when the static friction force is maximized, when $F = \mu_s N$. This is the maximum value of w so that the person does not slide up the wall. Substitute $F = \mu_s N$ into equation (2) above:

$$\Rightarrow -\mu_s N - mg\sin\theta = -m w^2 R\cos\theta$$

Combine this with equation (1) above to solve for w:

$$w = \sqrt{\frac{g(\sin\theta + \mu_s\cos\theta)}{R(\cos\theta - \mu_s\sin\theta)}}$$

For $g = 9.8\, m/s^2$, $R = 6\, m$, $\mu_s = 0.25$, $\theta = 70°$

$w = 3.95$ rad/s (maximum) (answer)

The range is $1.56 \text{ rad/s} \leq w \leq 3.95 \text{ rad/s}$