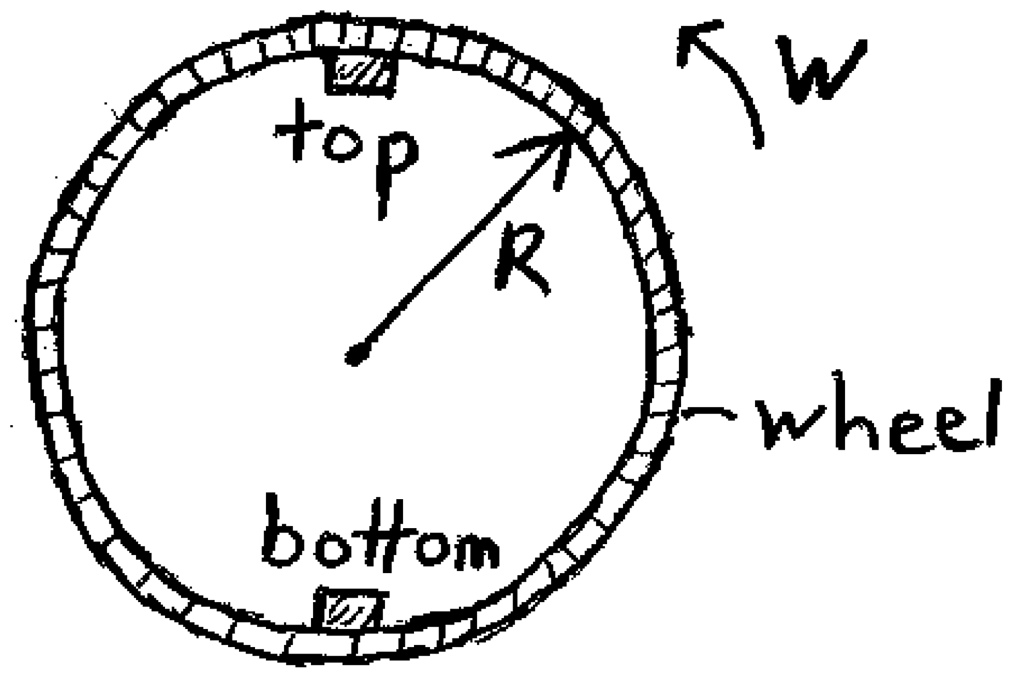
This is a force and motion problem involving uniform circular motion.



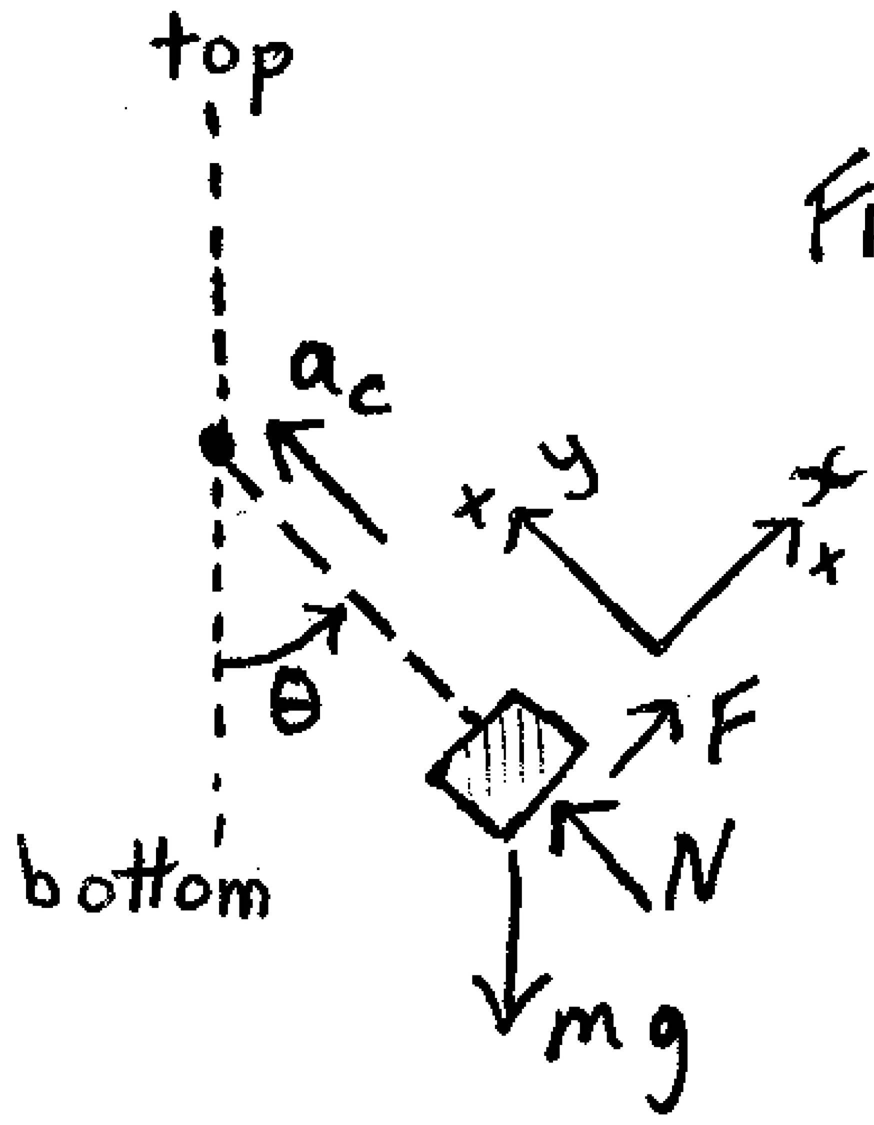

A block is sitting on the inside of a wheel rim, which has radius $R = 1.4\,m$. The wheel is oriented vertically and is rotating at an angular velocity $\omega$. The coefficient of static friction between wheel and block is 0.20.

(a) What is the minimum value of $\omega$ so that the block doesn't slide on the rim?

(b) What is the minimum value of $\omega$ so that the block doesn't fall off the rim?

(c) For each of the values of $\omega$ in (a) and (b), what is the contact force between the block and rim at the top and bottom position, as shown? Use mass of block $= 2.5\,kg$.

Solution:




Free-body diagram

Answer (b) First since it's easier.

Assume the block is rotating with the rim with no slipping. The block is most likely to lose contact with the rim at the top position, when $\theta = 180°$. Apply Newton's second law at this position, in the vertical y-direction:

$$\downarrow + \quad mg + N = ma_y \,, \quad a_y = a_c = w^2 R \quad \text{(centripetal acceleration)}$$

$$\Rightarrow a_y = a_c = \frac{mg + N}{m}$$

$$\Rightarrow w^2 R = \frac{mg + N}{m}$$

minimum $w$ occurs when $N = 0$ (on the brink of losing contact)

$$\Rightarrow w^2 R = g$$

$$\text{minimum } w = \sqrt{\frac{g}{R}} = \sqrt{\frac{9.8}{1.4}} = 2.65 \text{ rad/s} \quad \text{(answer)}$$

(a) For there to be no slipping, the maximum allowable static friction force must not be exceeded. This means that $|F| \leq \mu_s N$.

The wheel is rotating at constant $w$, so $a_x = 0$.

Apply Newton's second law in $x$-direction:

$$-mg \sin\theta + F = m(0)$$

$$\Rightarrow F = mg\sin\theta \qquad (1)$$

Apply Newton's second law in y-direction:

$$N - mg\cos\theta = ma_y \quad , \quad a_y = a_c = w^2 R$$

$$\Rightarrow N - mg\cos\theta = mw^2 R$$

$$\Rightarrow N = mw^2 R + mg\cos\theta \quad (2)$$

For there to be no slipping,

$$|F| \leq \mu_s N$$

Substitute equations (1) and (2):

$$mg\sin\theta \leq \mu_s (mw^2 R + mg\cos\theta)$$

$$\Rightarrow g\sin\theta \leq \mu_s (w^2 R + g\cos\theta)$$, valid for $0 \leq \theta \leq 180°$

Rewrite this:

$$\Rightarrow g\sin\theta - \mu_s g\cos\theta \leq \mu_s w^2 R$$

$$\Rightarrow g\frac{(\sin\theta - \mu_s\cos\theta)}{\mu_s R} \leq w^2$$

$$\Rightarrow w^2 \geq g\frac{(\sin\theta - \mu_s\cos\theta)}{\mu_s R}$$

For $180° \leq \theta \leq 360°$, the solution is the same as for $0 \leq \theta \leq 180°$

Substitute values:

$$\Rightarrow w^2 \geq 9.8\underbrace{\frac{(\sin\theta - 0.20\cos\theta)}{(0.20)(1.4)}}$$

The maximum value of this is 35.69 at $\theta = 101°$.

Therefore $w^2 \geq 35.69$

and $w \geq \sqrt{35.69} = 5.97\ \text{rad/s}$

This indicates that the minimum $w$ from part (b) is not realistic since the block would likely slide and fall off the rim before reaching the top $\Rightarrow$ minimum $w = 5.97\ \text{rad/s}$ (answer)

(c) Apply Newton's second law at the bottom position, $\theta = 0°$:

$$-mg + N = ma_y \ , \quad a_y = a_c = w^2 R$$

$$\Rightarrow N = m(g + a_y)$$

$$\Rightarrow N = m(g + w^2 R)$$

For part (a), $N = 2.5(9.8 + 2.65^2(1.4)) = 49.08\,N$
For part (b), $N = 2.5(9.8 + 5.97^2(1.4)) = 149.24\,N$
(answer)

Apply Newton's second law at the top position, $\theta = 180°$:

$$mg + N = ma_y \ , \quad a_y = a_c = w^2 R$$

$$\Rightarrow N = m(a_y - g)$$

$$\Rightarrow N = m(w^2 R - g)$$

For part (a), $N = 2.5(2.65^2(1.4) - 9.8) = 0\,N$
For part (b), $N = 2.5(5.97^2(1.4) - 9.8) = 100.24\,N$
(answer)