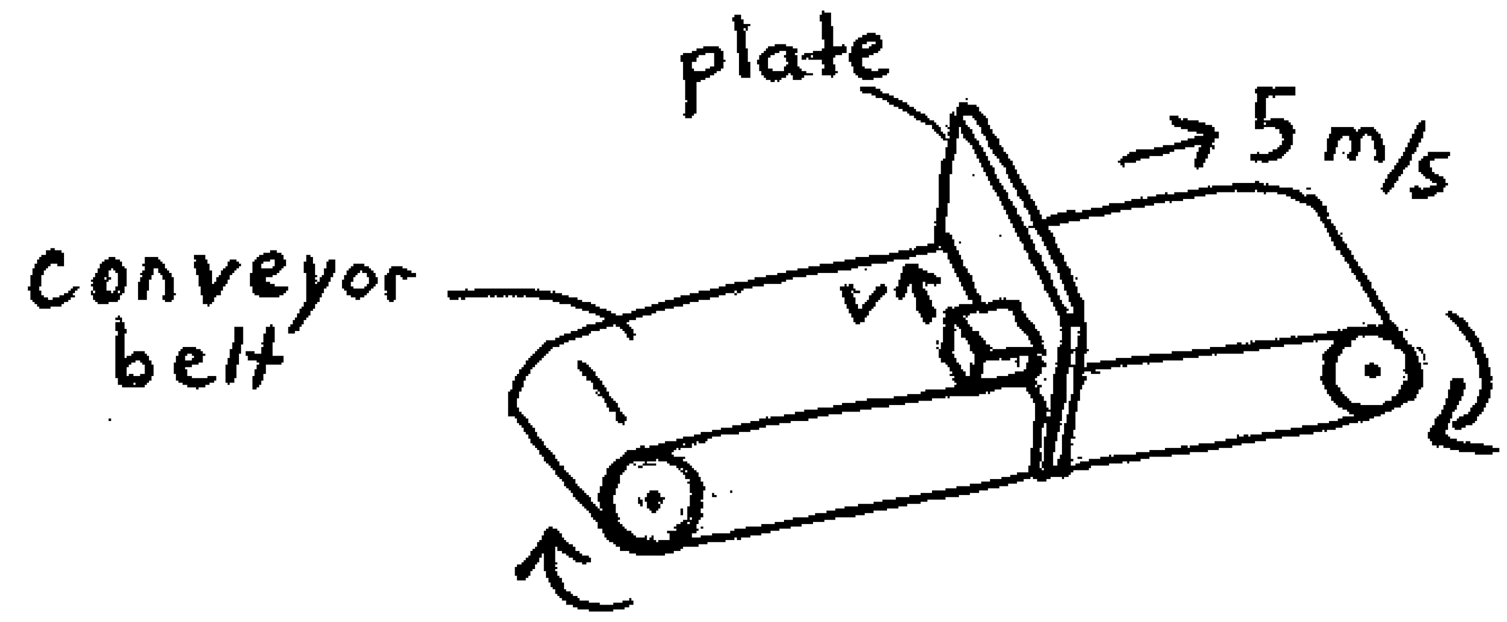
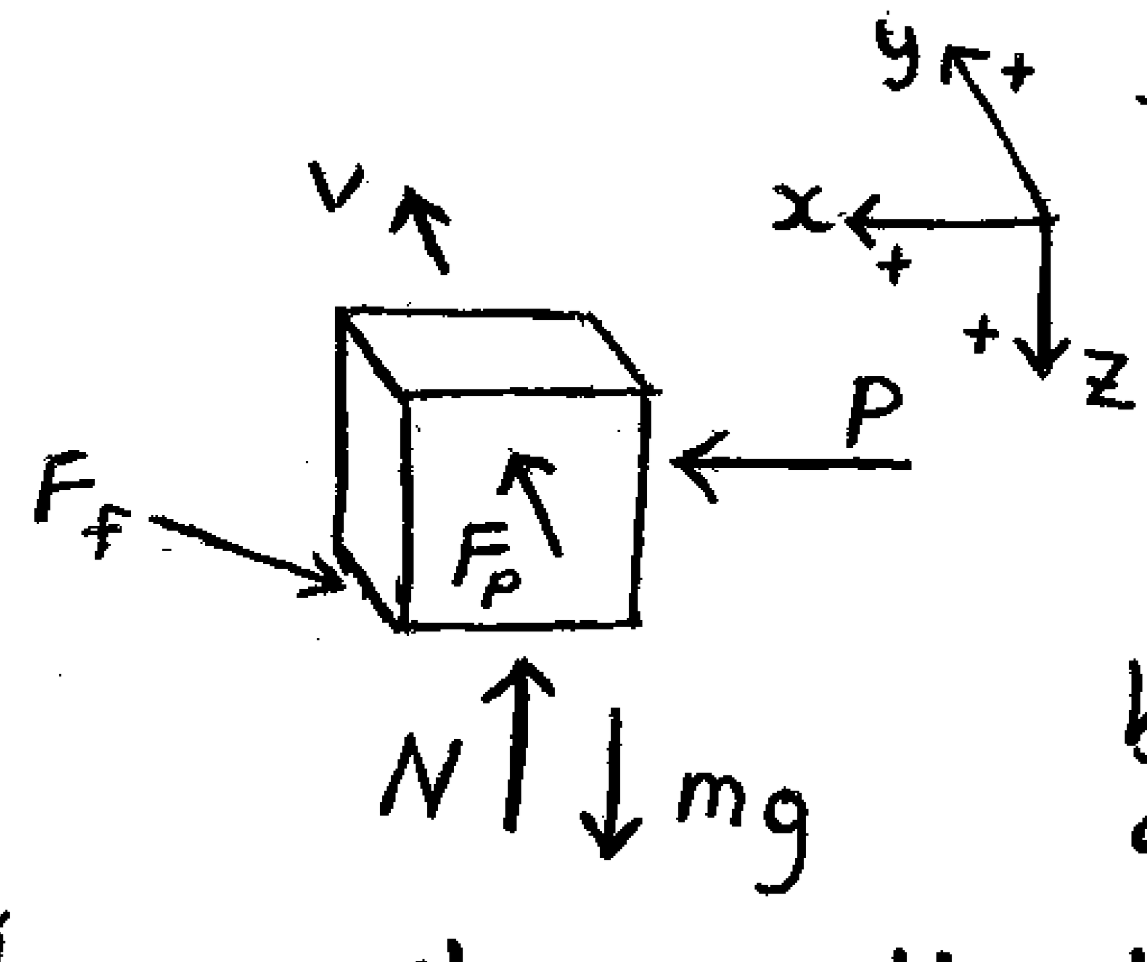
This is a force and motion problem involving friction.



A block of mass 2 kg is sitting on top of a conveyor belt moving at 5 m/s. The block is prevented from moving with the belt with a plate, as shown. As a result, the belt slides underneath the block with a speed of 5 m/s relative to the block. The coefficient of kinetic friction between the block and belt is 0.20, and there is no friction between the block and the plate. Suppose that someone were to push the block in the transverse direction at a speed v, as shown. What is the force applied by the person?

<u>Solution:</u>



F_f and N act on bottom face of block

P is the contact force between block and plate

$F_f = \mu_k N$ (This is the resultant friction force acting on block, which is constant)

F_p is the force exerted on block by person.

Find the velocity of the belt relative to the block:

$$\vec{V}_{block} = \vec{V}_{belt} + \vec{V}_{block/belt}$$

$\vec{V}_{block}$: relative to ground; $\vec{V}_{belt}$: relative to ground; $\vec{V}_{block/belt}$: velocity of block relative to belt

$$\vec{V}_{block/belt} = \vec{V}_{block} - \vec{V}_{belt}$$

$$\vec{V}_{block} = V\hat{j} \ m/s$$

$$\vec{V}_{belt} = -5\hat{i} \ m/s$$

So, $\vec{V}_{block/belt} = V\hat{j} + 5\hat{i} \ m/s$

$\hat{i}$ and $\hat{j}$ are unit vectors pointing along the positive x and y directions, respectively.

The Friction Force acts opposite the direction of $\vec{V}_{block/belt}$.

The friction Force is a vector:

$$\vec{F}_F = -\mu_K N \left(\frac{\vec{V}_{block/belt}}{|\vec{V}_{block/belt}|} \right)$$

unit vector pointing along direction of $\vec{V}_{block/belt}$

$N = mg$ (normal force acting on block)

$m = 2kg$

$\mu_K = 0.20 \quad g = 9.8 \ m/s^2$

$$|\vec{V}_{block/belt}| = \sqrt{v^2 + 5^2} = \sqrt{25 + v^2}$$

Then,

$$\vec{F}_f = \frac{-3.92 v \hat{j}}{\sqrt{25+v^2}} - \frac{19.6 \hat{i}}{\sqrt{25+v^2}}$$

Since the block is moving at constant velocity,

$$\vec{F}_p + \vec{P} + \vec{F}_f = 0$$ (in the plane of the conveyor belt, which is horizontal)

$$\Rightarrow \vec{F}_p + \vec{P} = -\vec{F}_f$$

$$\vec{F}_p = F_p \hat{j}$$

$$\vec{P} = P \hat{i}$$

Therefore,

$$F_p \hat{j} + P \hat{i} = \frac{3.92 v \hat{j}}{\sqrt{25+v^2}} + \frac{19.6 \hat{i}}{\sqrt{25+v^2}}$$

By comparison,

$$F_p = \frac{3.92 v}{\sqrt{25+v^2}}$$ (answer)

This is a hard problem because you really have to get at the heart of the concept of kinetic friction.