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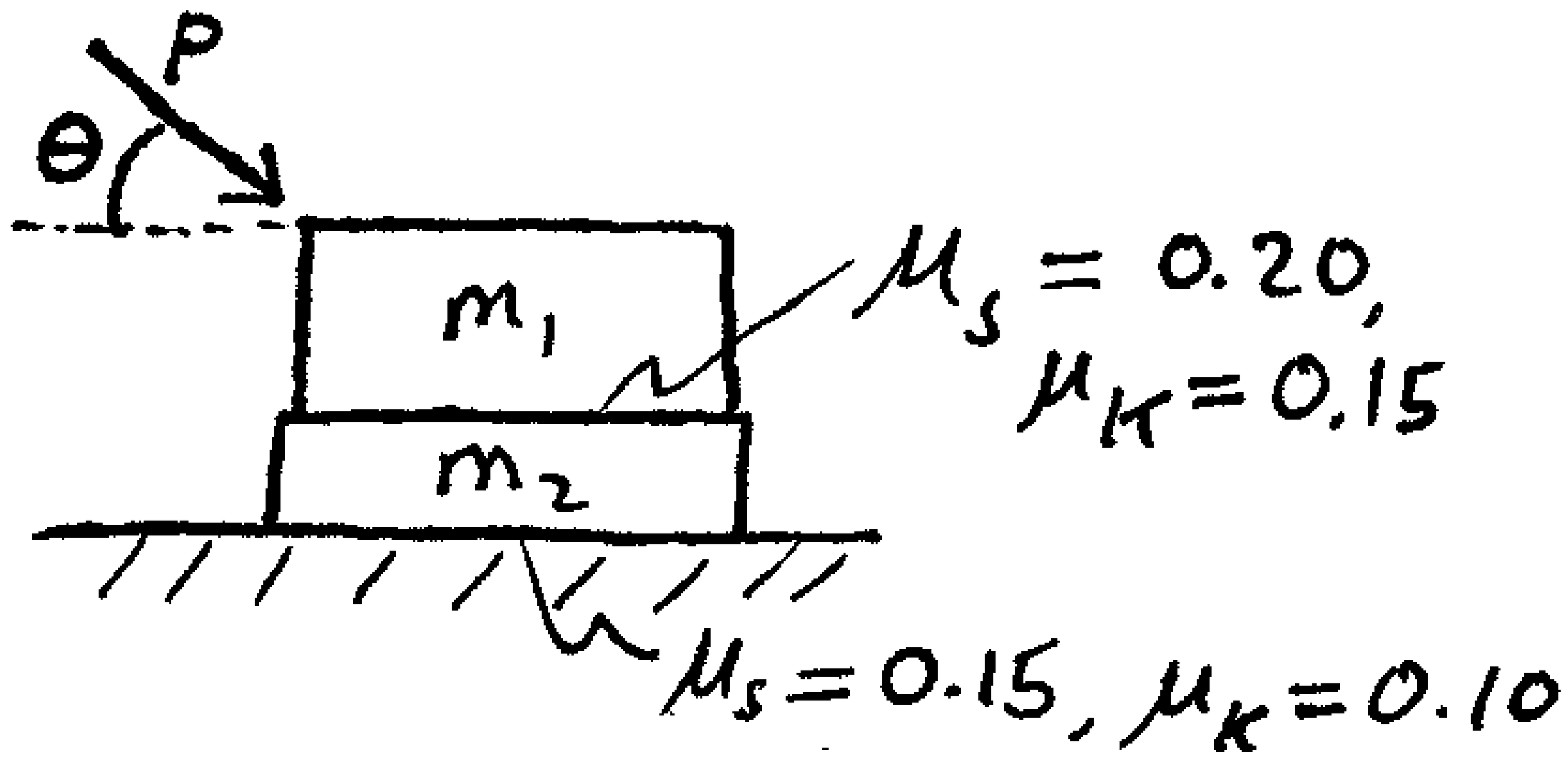
Oct. 1, 2020

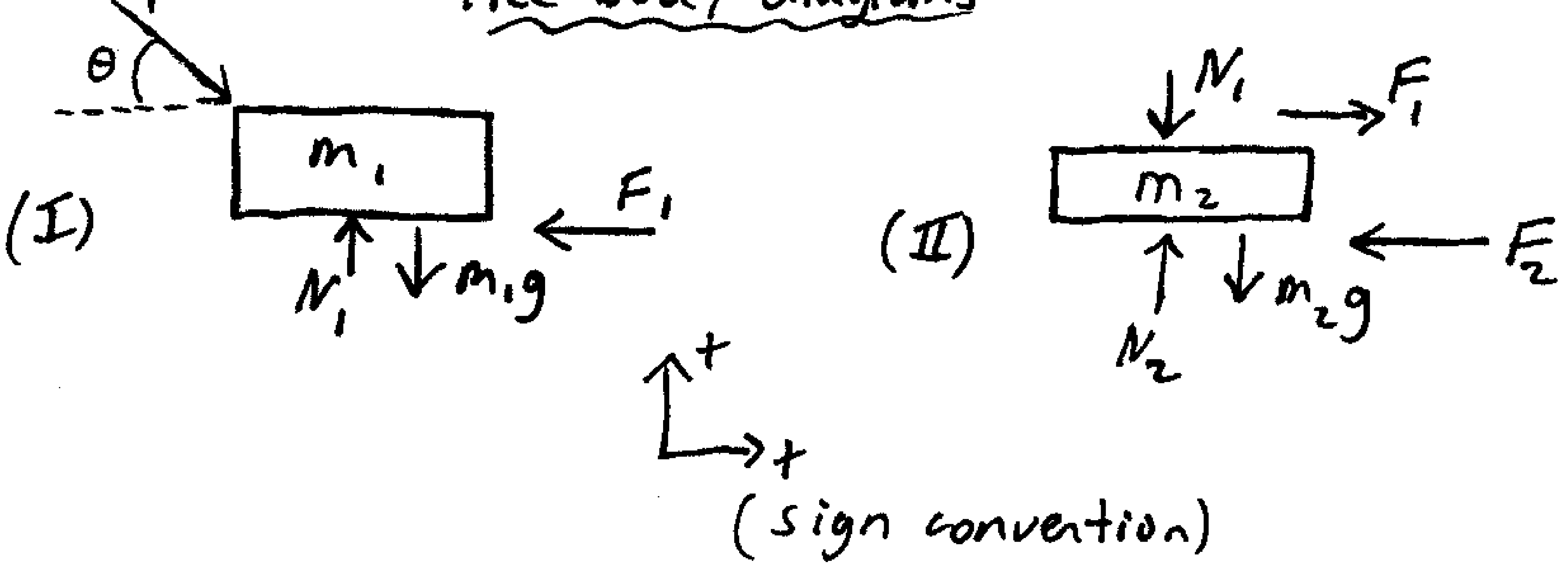
This is a force and motion problem involving friction.

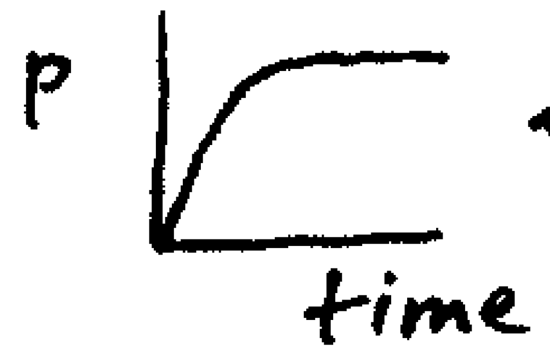


A force P is applied to a mass m_1 which makes an angle θ with the horizontal. Find an expression for $P(\theta)$ for the case where (a) the mass m_1 and mass m_2 move as a single unit, and (b) there is relative slipping between mass m_1 and mass m_2.

Solution:

Free-body diagrams





⟸ Assume P is not instantly applied and that there is a ramping up.

(a) This means the bottom block slips only, so that $P\cos\theta > \mu_s N_2$, $\mu_s = 0.15$ $\Rightarrow P\cos\theta > 0.15 N_2$ (1)

The blocks then move together at acceleration a.

Apply Newton's second law to the blocks moving as a single unit.

horizontal direction: $P\cos\theta - F_2 = (m_1 + m_2)a$

$$F_2 = \mu_K N_2 \quad , \mu_K = 0.10$$

$$\Rightarrow \quad a = \frac{P\cos\theta - 0.10 N_2}{m_1 + m_2} \quad (2)$$

There is no acceleration in vertical direction so,

$$-N_1 - m_2 g + N_2 = 0 \quad (II)$$

$$-P\sin\theta - m_1 g + N_1 = 0 \quad (I)$$

$$N_1 = P\sin\theta + m_1 g \quad (3)$$

and

$$N_2 = N_1 + m_2 g = P\sin\theta + (m_1 + m_2)g \quad (4)$$

Apply condition where there is no slipping between the blocks:

$$F_1 < \mu_s N_1 \quad , \mu_s = 0.20$$

(I) Apply Newton's second law to top block, in horizontal direction:

$$P\cos\theta - F_1 = m_1 a$$

$$F_1 = P\cos\theta - m_1 a$$

Then, $P\cos\theta - m_1 a < 0.20 N_1$ (5)

(for no slipping between blocks)

Lastly, apply condition where the bottom block slips first:

$$\mu_{s1} N_1 > \mu_{s2} N_2$$

$$0.20 N_1 > 0.15 N_2 \quad (6)$$

There are 3 inequalities which altogether constrain $P(\theta)$. They are:

$$(1) \Rightarrow P\cos\theta > 0.15\left(P\sin\theta + (m_1+m_2)g\right)$$

$$(5) \Rightarrow P\cos\theta - m_1 \frac{\left(P\cos\theta - 0.10\left(P\sin\theta + (m_1+m_2)g\right)\right)}{m_1+m_2} < 0.20\left(P\sin\theta + m_1 g\right)$$

$$(6) \Rightarrow 0.20\left(P\sin\theta + m_1 g\right) > 0.15\left(P\sin\theta + (m_1+m_2)g\right)$$

Solve in terms of P:

$$(1) \Rightarrow P > \frac{0.15A}{\cos\theta - 0.15\sin\theta}$$

$$A = (m_1+m_2)g$$

$$B = \frac{m_1}{m_1+m_2}$$

$$C = m_1 g$$

$$(5) \Rightarrow P < \frac{0.20C - 0.10AB}{\cos\theta - B\cos\theta + 0.10B\sin\theta - 0.20\sin\theta}$$

$$(6) \Rightarrow P > \frac{0.15A - 0.20C}{0.05\sin\theta}$$

(answer)

(b) This means the top block slips over the bottom block, so that

$$P\cos\theta > \mu_s N_1 \ , \ \mu_s = 0.20 \Rightarrow P\cos\theta > 0.20 N_1 \quad (7)$$

Apply condition where the top block slips first:

$$\mu_{s_2} N_2 > \mu_{s_1} N_1$$

$$0.15 N_2 > 0.20 N_1 \quad (8)$$

There are 2 inequalities which altogether constrain $P(\theta)$. They are:

$$(7) \Rightarrow \quad P\cos\theta > 0.20(P\sin\theta + m_1 g)$$

$$(8) \Rightarrow \quad 0.15(P\sin\theta + (m_1+m_2)g) > 0.20(P\sin\theta + m_1 g)$$

Solve in terms of P:

$$(7) \Rightarrow P > \frac{0.20C}{\cos\theta - 0.20\sin\theta} \qquad A = (m_1+m_2)g$$

(answer)

$$(8) \Rightarrow P < \frac{0.15A - 0.20C}{0.05\sin\theta} \qquad C = m_1 g$$

Note: the bottom block may or may not slip, but that's irrelevant here.

This is a very hard problem!