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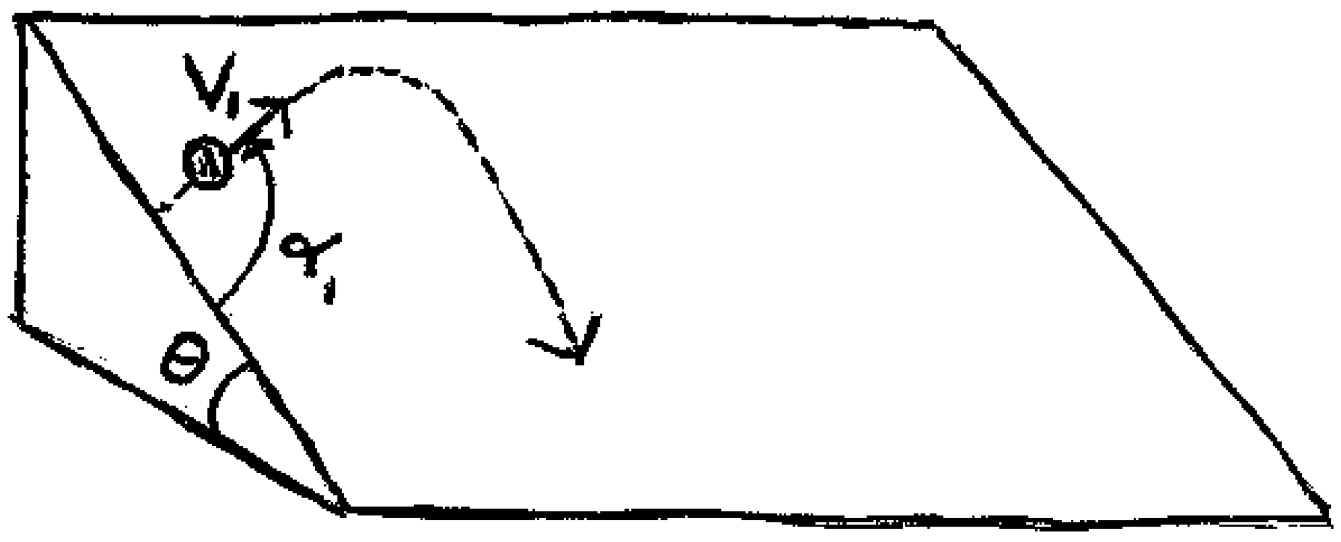
Sep. 25, 2020

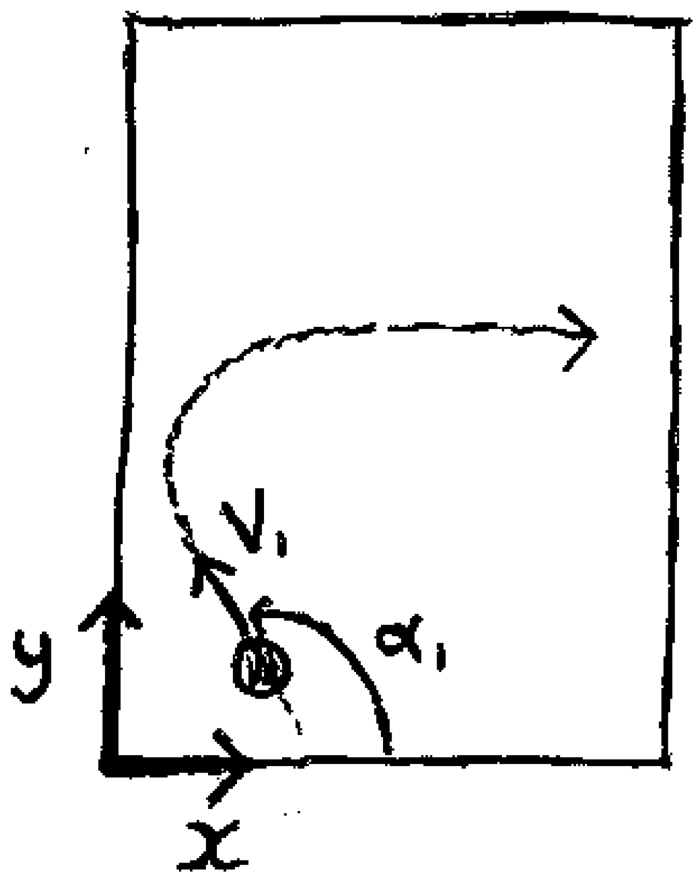
This is a force and motion problem involving friction.



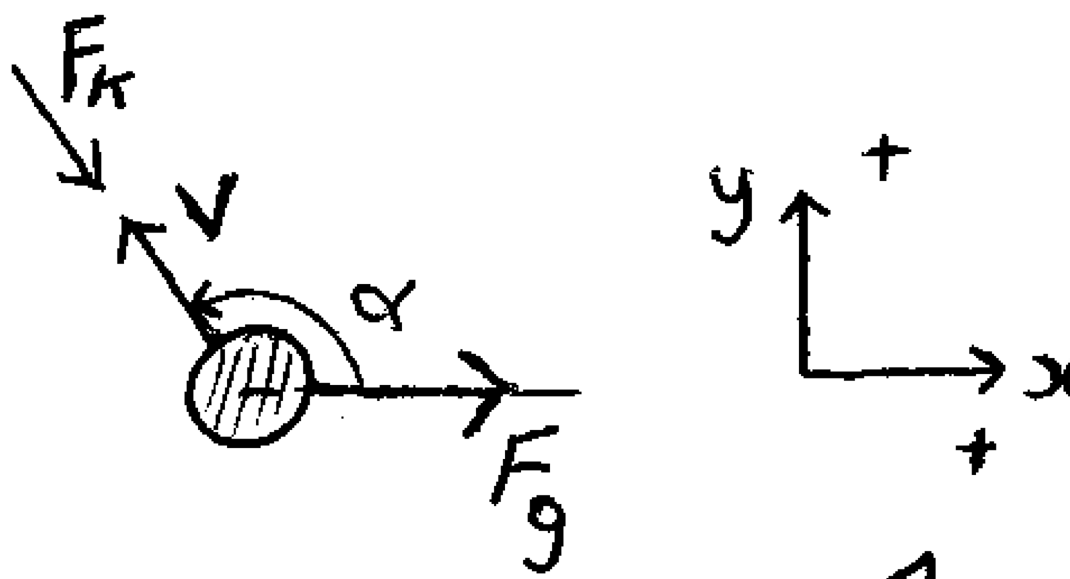
A disk is launched at an initial speed V_1 at an angle α_1 as shown, on an inclined plane. If the coefficient of kinetic friction is equal to $\tan\theta$, what is the final speed of the disk assuming the incline is large enough so that the disk slides for a long time?

Solution:

This is an adapted problem from a physics contest. The solution to this is very non-obvious. Preparation with similar problems is the only way to have a reasonable chance of solving it during a test (or physics contest).



Free-body diagram



The plane of xy is in the same plane as the incline.

$\Rightarrow F_g = mg\sin\theta$ (gravity force in direction of incline)

$F_K = \mu_K N$

$\mu_K = \tan\theta$

$N = mg\cos\theta$ (normal force acting on disk)

$\Rightarrow F_K = mg\sin\theta$

Apply Newton's second law in x-direction:

$$-F_K \cos\varphi + F_g = m a_x$$

$$\Rightarrow -mg\sin\theta\cos\varphi + mg\sin\theta = m a_x \quad (1)$$

Apply Newton's second law along the direction of motion of the disk, where the direction of motion is treated as the positive direction:

$$-F_K + F_g\cos\varphi = m a$$

(a: component of acceleration along direction of motion (this acceleration is tangent to the disk's path))

$$\Rightarrow -mg\sin\theta + mg\sin\theta\cos\varphi = ma \quad (2)$$

Add equations (1) and (2):

$$0 = m a_x + m a$$

$$0 = a_x + a$$

$$0 = \frac{dv_x}{dt} + \frac{dv}{dt} \rightarrow \frac{d}{dt}(v_x + v) = 0$$

So, $v_x + v = \text{constant}$

(v is the velocity of the disk, which is tangent to the disk's path.)

At long time, $v_x = v = v_F$ (final speed)

So, $2v_F = \text{constant}$

Initially, $v_x = v_1\cos\varphi_1$, and $v = v_1$

Therefore, $v_1\cos\varphi_1 + v_1 = 2v_F$, and $v_F = \dfrac{v_1(1+\cos\varphi_1)}{2}$

(answer)