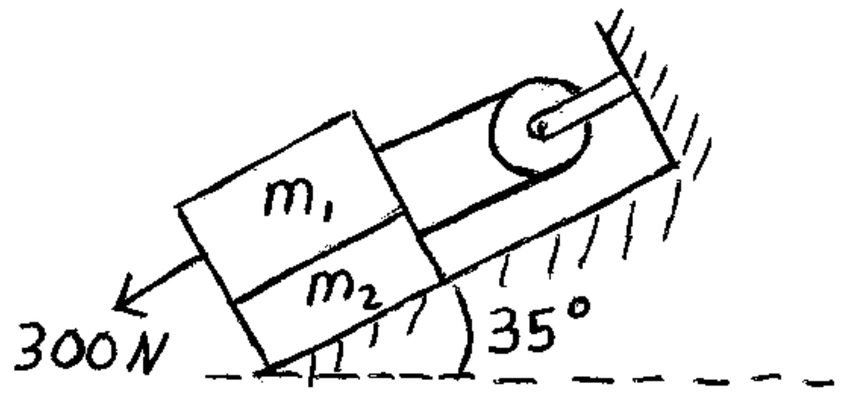
Sep. 24, 2020

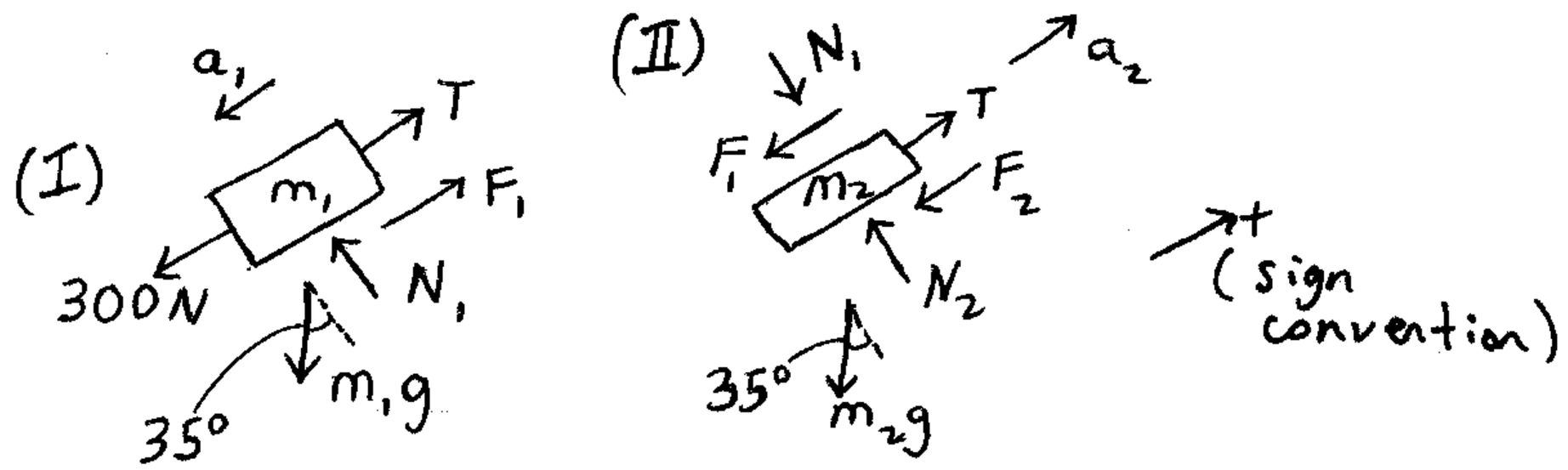
This is a force and motion problem involving friction.



In the system shown, the coefficient of static friction between all contact surfaces is 0.15, and the coefficient of kinetic friction between all contact surfaces is 0.10. Determine the acceleration of the blocks and the tension in the rope. Note that $m_1 = 20\,kg$ and $m_2 = 10\,kg$.

Solution:

Free-body diagrams



First check if the system is able to remain in static equilibrium due to static friction.

(I) $T + F_1 - m_1 g \sin 35° - 300 = 0$ (1) ($\sum F = 0$ along direction of incline)

(II) $T - F_1 - F_2 - m_2 g \sin 35° = 0$ (2)

Eliminate T between (1) and (2):

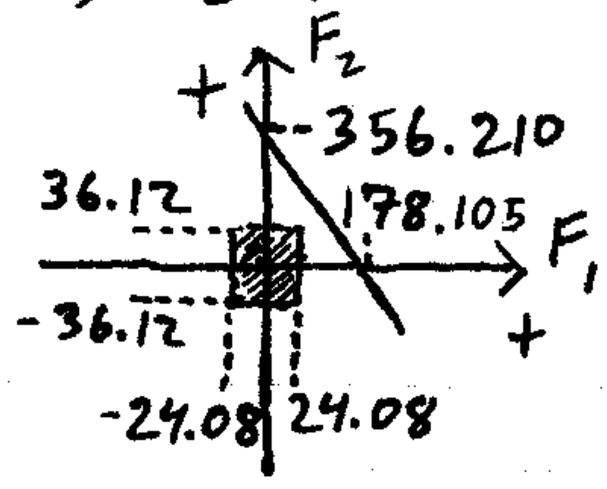
$$300 + m_1 g \sin 35° - F_1 = F_1 + F_2 + m_2 g \sin 35°$$

$$\Rightarrow \quad 2F_1 + F_2 = 356.210 \quad (3)$$

The range of static friction force between the two blocks is $-\mu_s N_1 \leq F_1 \leq \mu_s N_1$. Now, $\mu_s = 0.15$ and $N_1 = m_1 g \cos 35°$, so $N_1 = 160.55 N$. So, the range is $-24.08 N \leq F_1 \leq 24.08 N$

Similarly, the range of statre friction force between the bottom block and the incline is $-\mu_s N_2 \leq F_2 \leq \mu_s N_2$. Now, $\mu_s = 0.15$ and $N_2 = (m_1 + m_2) g \cos 35°$, so $N_2 = 240.83 N$. So, the range is $-36.12 N \leq F_2 \leq 36.12 N$

Draw the graph of equation (3) in the F_1, F_2 plane, which is a straight line:



Static equilibrium is only possible for F_1 and F_2 inside the shaded region, where static friction force is able to maintain static equilibrium. Since the line for equation (3) is outside the shaded region. Then static equilibrium is not possible, and slipping occurs. This then becomes a dynamics problem.

(I) Apply Newton's second law along direction of incline:

$T + F_1 - m_1 g \sin 35° - 300 = m_1 a_1$, $F_1 = \mu_K N_1$ $(\mu_K = 0.1)$

(II) Apply Newton's second law along direction of incline:

$T - F_1 - F_2 - m_2 g \sin 35° = m_2 a_2$, $a_2 = -a_1$, $F_2 = \mu_K N_2$

Combine above two equations and solve:

$a_1 = -10.0\ m/s^2$, $a_2 = 10.0\ m/s^2$, $T = 196.35\ N$

(answer)